REVERSE ENGINEERING UTILIZING DOMAIN SPECIFIC KNOWLEDGE

by

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SUPERVISORY COMMITTEE APPROVAL

of a dissertation submitted by

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This dissertation has been read by each member of the following supervisory committee and by majority vote has been found to be satisfactory.

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I have read the dissertation of ________H. James de St. Germain_______ in its final form and have found that (1) its format, citations, and bibliographic style are consistent and acceptable; (2) its illustrative materials including figures, tables, and charts are in place; and (3) the final manuscript is satisfactory to the supervisory committee and is ready for submission to The Graduate School.

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ABSTRACT

Reverse engineering is the process of defining and instantiating a model based on the measurements taken from an exemplar object. The measurement (or data sensing) process is prone to random and systematic errors and often fails to sense the object in a manner consistent with the intended functionality of the object's design. Therefore a model fit directly to the data will not faithfully capture the geometry of the part (the form), nor the relationships among features of the part (the function) as originally specified by the designer.

Manmade objects are often well defined, following specific rules and structures based on perceived pragmatics. This is especially true in the case of mechanical two and a half dimensional (2.5D) machined parts. Because of the high accuracy needs of this domain, reverse engineering techniques using generic primitives are inappropriate. This dissertation asserts that an understanding of common design practices and manufacturing knowledge specific to 2.5D machining can and should be used to guide the reverse engineering process in order to achieve higher accuracy models.

To this end, reverse engineering is characterized as a constrained optimization problem. Logical laws of form are encoded as constraints in order to coerce new models to emulate the structure common to this genus of parts while best approximating the sensed data. A technique has been created to automatically hypothesize likely constraints that should hold on a hypothesized model. These constraints drive a DOF reduction process on the model and are further encoded as penalty functions during the model optimization. The entire process is formulated in a manner consistent with modern optimization techniques.

For Dad

(and you too Mom)

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CHAPTER 1

INTRODUCTION

Reverse engineering, defined as model creation from exemplar[1] objects, is becoming more prevalent in many industries, including manufacturing, automotive design, and computer animation. While fields such as computer animation can often get by with low resolution models enhanced with texture maps, such fields as automotive design require high precision models in order to merge assemblies of parts and check their interaction based on their real world physical properties. This work focuses on techniques for recreating high accuracy models of manufactured mechanical parts, specifically those designed for 2.5 axis milling.

Mechanical 2.5D machined parts[2] are defined by their 2D profiles in the X-Y plane and an extrusion depth. These parts are typically created using a three axis machining center. Such parts are found in many real world applications and represent a rich and complex geometric environment, yet one that has sufficient structure that can be exploited in the reverse engineering process. Further, these parts are often difficult to measure precisely using optical or manual scanning techniques because of their reflectance patterns, abrupt discontinuities, curved surfaces, and deep concavities.[3]

The traditional solution is to have an engineer take the part, make sketches of it, measure it using handheld calipers, and recreate a new model representing his or her

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[1] The term exemplar is used to refer to the physical object of interest.

[2] Throughout the dissertation, the terms part, exemplar, and object will be used interchangeably to refer to mechanical parts designed for and machined on three axis milling machines.

[3] In fact, it is often the case that the sensed data is poorest in the areas where the most accuracy is needed, such as feature intersections.

interpretation of the part. Although calipers measure certain widths and lengths accurately, they unfortunately are often not appropriate for measuring relationships between features of the object nor do they easily handle sculptured surfaces. Thus the expert's own judgment has to be used to correct for the lack of accurate measurements of the physical properties under consideration. This makes for a time-consuming process and often results in a redefinition of the model, not a replication.

Modern sensing-tools (i.e., vision systems, laser systems, and touch probes) are capable of providing copious amounts of data not easily acquired using manual calipers. This data is processed using low-level generic primitives[4] to fit a wide variety of topological domains from solely free-form geometry to well-structured manufactured parts. The strength and weakness of these systems is their generality. Although such systems can produce a model for "any" surface, they often fail to achieve the required accuracy, and they do not capture the functionality behind the objects they represent. Current computerized geometry reconstruction systems are not sufficient for overcoming the noise associated with the sensed data, nor do they produce appropriate models necessary for redesign or manufacture. This limitation is rooted in two main causes: sensor error and model inappropriateness.

Although many data sensing techniques exist, all suffer from different problems and inaccuracies, resulting in random and systematic noise in the data. This work uses data produced by an automatic laser range finder. Such data present a cloud of 3D points that lie within an error bound of the true surfaces. This error bound is often larger than the required accuracy of the model application. Further, these data clouds are unstructured, often having missing areas of data, and containing data with systematic errors.

The second reason current methods can fail is that they are inappropriate to the specific goals of a given type of reverse engineering. For example, a triangulated mesh perfectly interpolating the sensed data, or a simplified mesh approximating the surface of the data, can be constructed. In either case, the representation fails to capture the

[4] Generic primitives include both triangulated mesh representations and simple geometric primitives such as spheres, cubes, and cylinders.

semantics associated with the geometry and functionality of many parts where topologically uniform surfaces are encountered. Likewise a spline patch approximating the geometry of a region does not embody the ontological fact that the geometry was actually a plane, or a hole, or some other mechanical feature.

1.1 Thesis

This work addresses creating appropriate and faithful models for mechanical 2.5D machined parts, reconstructed from physical exemplars, that maintain tight tolerances compared to the original part as designed. The success of this effort has been in the ability to incorporate domain specific knowledge of manufacturing and design in such a way as to produce new models that are truer to the original design.

This dissertation proposes a new method for mapping domain-specific knowledge about manufacturing and design into the reverse engineering process. Knowledge is formulated in terms of geometric and parametric constraints that confine the topology of the hypothesized model to well-defined forms. The reverse engineering process is defined as an optimization framework attempting to minimize the distance from the sensed data to the hypothesized model while enforcing all constraints. The constraints directly and indirectly impose structure on the geometry of the model and are mathematically formulated in a manner that allows them to be integrated into global optimization methods. The process produces CAD models that more accurately represent the form and function of the exemplar parts as originally designed.

This research demonstrates the effectiveness of constraint-based reverse engineering in overcoming sensor error and achieving faithful and accurate models. Faithful models logically depict the objects as they might have been designed and attempt to use a minimal set of parameter to do so. Accurate models contain little positional error when compared to the geometry of the original design.

A set of constraints representing knowledge of 2.5 axis milling and design has been proposed. These constraints, as well as the geometry they apply to, have been formulated for optimization using standard optimization packages. Finally, a method for automatically hypothesizing likely constraints has been created. This dissertation makes no attempt to advance the core area of optimization, but instead details how reverse engineering/model reconstruction can be formulated in a manner that takes advantage of the strengths of current optimization strategies.

1.2 Goals and Validation Techniques

The operational domain of this dissertation is that of mechanical part re-creation, and thus the primary application-area goal is to create high precision models from exemplar parts that reflect not only the geometry of, but as importantly, the design intent behind the geometry. The pedagogical goal behind this research is to show that knowledge in this domain can be encoded directly using geometric and parametric constraints in a manner fit for use with standard optimization algorithms. The result is a reverse engineering method couched in the terms of constrained optimization that produces models that are more accurate than previously attainable.

A new computational framework has been created which represents design and manufacturing knowledge as constraints that guide a global optimization process. Additionally, a technique is advanced for automatically hypothesizing and asserting constraints based on such knowledge. Finally a system has been developed which encompasses the reverse engineering process, rapidly and semi-automatically segmenting and fitting unordered data clouds, hypothesizing constraints, optimizing based on the constraints and data, and finally producing faithful, high accuracy CAD models.

To validate the claims of this dissertation, exemplar parts and designs were taken from mechanical parts created and designed for automotive systems. The initial models were available for comparison with the reverse engineered models. The results both in terms of global error measurements and in terms of parametric faithfulness to the original design are given for a variety of exemplar parts. These results show that the techniques developed for this research are effective and powerful.

1.3 Reverse Engineering Process

The model creation process, as applicable to this work, is shown in Figure 1-1. A 2.5D part, for which a new model is required, is scanned by a laser range finding sensor to produce a representative set of 3D points.

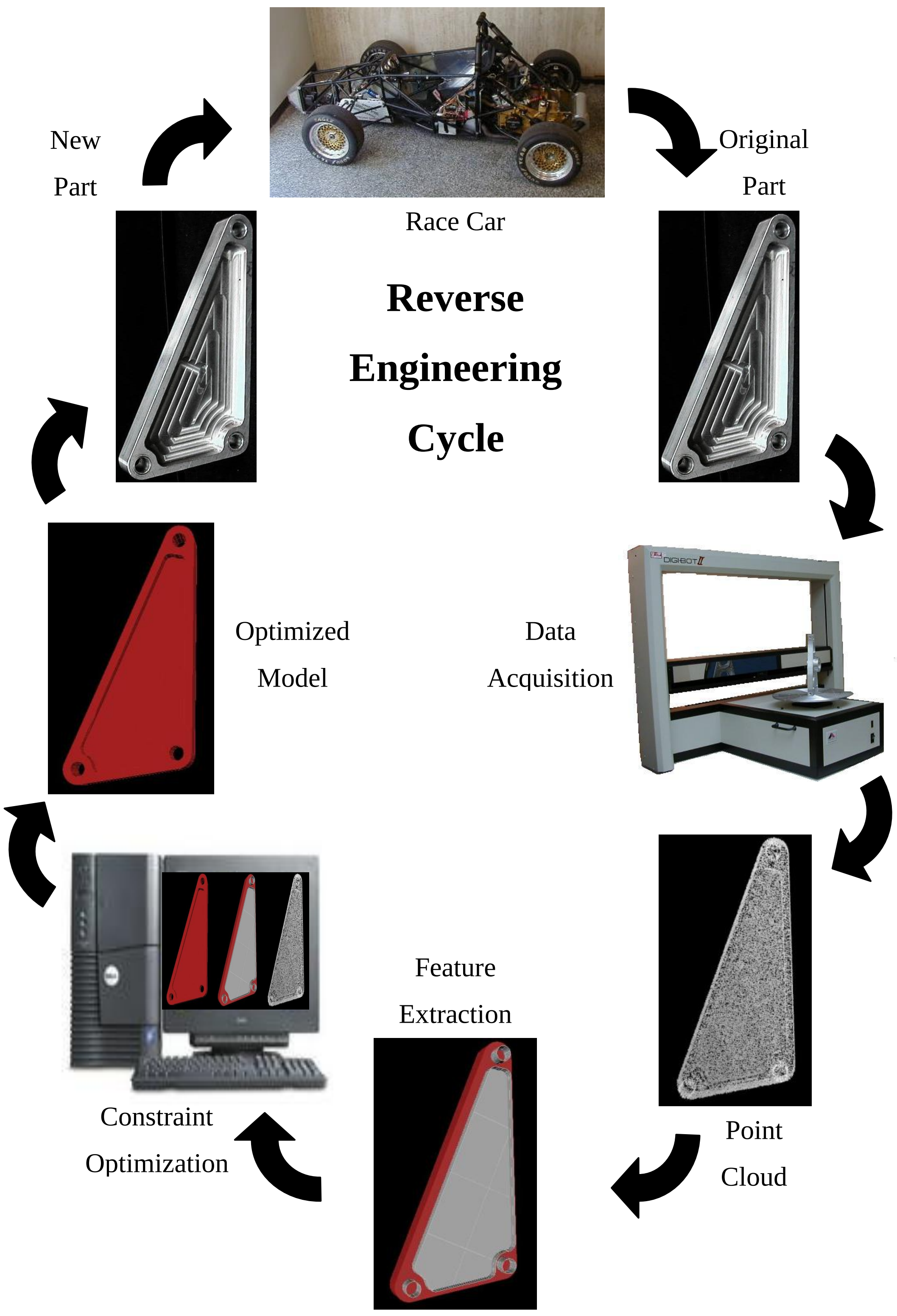

Figure 1-1: The Reverse Engineering Process

The points are partitioned based on local areas of common geometry. They are then fit to an initial set of geometric primitives. This initial model is analyzed to produce a list of likely constraints on the geometry that are deemed to hold based on design principles. An optimization procedure is then applied to produce a new model that corresponds to the 3D points but maintains certain high level properties. The resulting model has increased accuracy over previous techniques and is in a form that can be re-engineered or directly machined.

1.4 Artifacts

Manmade objects are almost always designed with far less than the total representational power of free-form geometry. In the case of mechanical 2.5D machined parts, the geometry usually contains many well-defined geometric relationships. Such artifacts include parallel planar faces, aligned holes, symmetric pockets, and common widths and radii. Such knowledge can be encapsulated as geometric and parametric constraints. Constraints restrict the possible geometric structures of the model during the optimization process to those that mimic the physical properties encountered in the exemplar object. Thus constraints can be viewed both as a language that allows an engineer to discuss geometric properties and as a tool that mathematically restricts the geometry to certain shapes.

"Modeling accuracy depends on effective use of properties that distinguish the geometry of interest from effects due to sensor noise" [7]. This research has identified three levels of constraints that are useful in representing the progression of knowledge used in model creation: domain specific primitives, domain specific pragmatics, and functional constraints. Domain specific primitives narrow the possible shape of the reconstructed model from arbitrary geometry down to a well-defined set of design and manufacturing features. Domain specific pragmatics attempt to capture specific geometric conditions and conformities that are likely to be found based on how a part is designed and manufactured. Functional constraints describe likely interaction among the features of the object.

Each level represents a broader view of how design artifacts are predicted by analyzing design intent. By progressively utilizing and enforcing each level of constraints, more knowledge is brought to bear on the problem creating more accurate models when compared to the original design.

1.5 Optimization Over Constraints

An optimization method can be interpreted as a process to minimize some undesirable criteria. In the unconstrained case, the optimization criterion is the geometric distance between the hypothesized model and the sensed data points. In the constrained case, the hypothesized model is created using a limited set of appropriate geometric forms that are then optimized based on the data, subject to certain geometric constraints. It is important to understand that the topological and geometric constraints asserted during the reverse engineering process prohibit the model from simply conforming to the sensed data, because it is known that the sensed data is only an approximation of the true form of the object.

To employ a compatible optimization method, the constraints and models must be represented mathematically. For the purposes of this work, the models and constraints are expressed in terms of symbolic parametric notation and geometric construction algorithms. This makes it possible to define error metrics for the sensed data and for the violation of constraints. This further allows the model to be redefined using fewer variables; this process is known as the symbolic degree of freedom (DOF) reduction process and is driven by the asserted constraints.

It is important to note that previous efforts on model reconstruction fit each feature (model element) of the object individually without consideration to its global function. Constraints accord a method to integrate multiple geometric features during a single optimization session, rather than as several distinct optimizations. This provides a powerful tool for accurately extracting the intended relationships and geometry over the entire exemplar object.

1.6 Overview

Chapter 2 details the background work that forms the foundation for constraint-based reverse engineering. This includes a review of reverse engineering considering both vision-based techniques and those dealing with pure range data. Further, a brief introduction is furnished for the field of optimization.

Chapter 3 contains a motivation for and discussion of the domain specific knowledge applied in building the constraint optimization framework. A set of design constraints is given as well as the motivation behind the use of these constraints for reverse engineering.

Chapter 4 describes how the constraints and geometry are formulated to work with the optimization methods. Details of the constructive geometry specific to this domain are shown and methods are described for reducing the complexity of models by asserting constraints to reparameterize and reduce the DOFs of the model.

Chapter 5 details the method for automatically hypothesizing constraints based on initial geometric and parametric fittings of the data.

Chapter 6 discusses the entire reverse engineering process and the resulting models. Quantitative results for each exemplar object are exhibited along with a qualitative description of how well the constraint assertion and optimization process functioned in recreating the object relative to how the designer likely envisioned it.

CHAPTER 2

BACKGROUND

The research presented here is based on many fields: reverse engineering techniques in the field of manufacturing, modeling, data segmentation and fitting, dimensioning and tolerancing, feature-based design, geometric constraint systems, and optimization theory.

2.1 Reverse Engineering in the Field of Manufacturing

Reverse engineering is the process of accurately duplicating an object (in many cases by creating a CAD model for the object). This process has found use in the areas of computer graphics, animation, medicine and CAD/CAM[5], among others. The need for reverse engineering in the field of manufacture has becoming increasingly important. A few common scenarios follow:

- Designers, such as in the automotive industry, sculpt new models from clay. CAD representations of these models are then required to produce the finished part.
- Spare parts are needed, but no CAD models or design processes exist for the part due to the part's antiquity or other business related reasons.
- Original CAD models no longer represent the true part because of subsequent undocumented modifications made after the initial design stage.[6]

[5] CAD/CAM represents computer aided design and manufacturing.

[6] Modifications are often introduced during the lifetime of a part, occurring as early as the initial manufacturing process when changes are sometimes made directly on the shop floor to facilitate the machining of the part.

Traband et al. [37] identify many of the concerns and opportunities that are now or will soon be associated with reverse engineering. They define the results of a reverse engineering operation as producing a type three drawing set and a set of intelligent CAD models of the components. Further, they define the reverse engineering preprocess as:

1. Collecting all available information and documentation, including nonproprietary drawings, functional requirements, tooling and fixturing requirements, processing and material requirements, etc.
2. Identifying new data elements required for a complete technical data package.
3. Performing a cost/benefit analysis.
4. Contacting the cognizant engineer.
5. Establishing a reverse engineering management plan.
6. Establishing acceptance criteria.

Once this process has been accomplished, the technical issues of the actual reverse engineering process must be addressed. This process usually starts by scanning[7] the part in question. The result of the scanning process is often simply a set of 3D geometric points associated with the surface of the object. Numerous early researchers from the vision community have reported on employing vision-based systems (using intensity and range image analysis) for reverse engineering. Specific examples of reverse engineering research can be found in [11], [18], [19], [22], [24], [25], [28], [30], [36], and [37] among others. These works form a foundation for modern forays into reverse engineering. Unfortunately, none provide the accuracy and fidelity needed for CAD models in the realm of mechanical parts.

Broacha and Young [4] describe the commercial state of the art of reverse engineering. They mention several factors that should be provided in any reverse engineering system. These include a facility for data import and export, a mathematical foundation of surface modeling, comprehensive functionality for displaying and manipulating point data, and the actual process for reverse engineering or surfacing. These criteria, along with those established above, greatly reduced the number of viable

[7] A list of scanners/digitizers can be found in [4]; information on coordinate measuring machines can be found in [28].

reverse engineering systems in either commercial or academic use. The constraint-based optimization approach described in this dissertation offers a path for adding domain specific knowledge into such a system.

2.2 Modeling

A geometric model represents the spatial aspects of an object. In the field of manufacturing, the traditional model is a blueprint, or engineering drawing. This "model" specifies geometric information as well as material, assembly, and tolerance information. It would be up to a manufacturing engineer to translate the model (drawing) into a manufacturing plan. In some cases, the manufacturer modifies the original drawing to simplify or facilitate the manufacturing process.

With the advancement of electronic technology, it was only natural to computerize the design and modeling stage, and even the manufacturing process itself. According to Dierckx [9] a good model should provide the following functionality:

- Parameter Estimation: When modeling a known curve, it is often necessary to instantiate the parameters of that curve.
- Functional Representation: Functions give us values over the entire range of the data, as well as provide derivative information.
- Data Smoothing: Because sensed data is subject to error, it is not sufficient nor desirable to interpolate the data, but rather it is necessary to approximate the true curve.
- Data Reduction: Storage, manipulation, and reasoning needs often require that large volumes of data be replaced by a set of parameters much smaller than the original data set.

The value of a computerized model lies in the ability of the computer to manipulate and reason about the model. A good model captures the crucial information necessary to construct or utilize an object while abstracting away erroneous detail. The computerized model construction process is similar to the original drafting process.[8] A

[8] See Sturgill [33] for a discussion on the necessity of "drawing" in the design process.

well-defined CAD system mimics the way an engineer designs a part, unobtrusively restricting the part definition to conform to well known design principles. The result is a CAD model that implicitly and explicitly describes the object as intended.

CAD models should replace the engineering drawings with all required information, including geometric and tolerance information, as well as retain the intent of the designer. It should further be possible to generate engineering drawings from the CAD model. Few modeling methods satisfy all these criteria.

Among the most often encountered computer geometry representational techniques are point clouds, spatial occupancy representations, B-Reps, Constructive Solid Geometry (CSG), generalized cones, polygonal meshes, and feature-based models.

Point clouds are collections of 3D geometric points that are associated with the surface of an object. Point clouds are the standard output of traditional sensing devices. They represent optically gathered or physically touched locations on the surface of real world objects and exhibit a tolerance range of deviation from the true surface. These clouds are used as the input to various data fitting algorithms. Point clouds are straightforward to maintain and manipulate as a single entity but are seldom used in the design process. They are cumbersome and lack precise geometric information to describe accurately all but the coarse shape of a part.

Spatial occupancy representations divide $\Re 3$ space into discrete and uniform subregions that can be combined to describe the volume enclosed by a part. Various representations are discussed by Besl [1] including voxel, octree, tetrahedral cell decomposition, and hyper-patch representations. Such a representation is useful for computing simulations of the properties of a part, but requires copious amounts of memory to store complex shapes and is not directly amenable to engineering processes.

Surface boundary representations, or B-Reps, are a modeling form in which "an object is modeled by a graph corresponding to a hierarchy of topological entities (faces, edges, vertices)" [32]. Such a model easily captures discontinuity information, but does not represent high level features (such as pockets or holes) or the design intent behind the model. B-Reps are often used to generate code that controls numerically controlled machining devices.

CSG models are formed by taking geometric primitives, such as rectangles or cylinders and combining them by invoking regularized Boolean operators, such as "and" and "or" [32]. A model is typically comprised of a large number of simple shapes that are added or subtracted (i.e., “Booleaned”) together hierarchically to define complex geometry. CSG models must be recomputed every time a primitive is moved to reevaluate edges and vertices of the object, and they cannot easily represent many manufactured objects.

Generalized cones describe an object as the area produced by sweeping an arbitrary 2D curve, or cross section, along a 3D space curve, known as the axis [1]. These shapes perform well in many situations, but are not general purpose enough to represent a rich set of free form surfaces.

Polygonal meshes, or wire-frame models, are simple boundary representations of data. They allow shaded renderings of objects and can be used in some simple tasks where deforming a complex object is not necessary. Meshes are also one form of model that can easily be constructed from actual data. As discussed in the next section, Hoppe et al., [16] suggest the mesh as a generalized surface reconstruction representation. A chief drawback to meshes is that they are piecewise linear representations of a model; most models require higher DOFs to accurately represent the true geometry. Further, the use of meshes in the design process can be rather cumbersome after only a few vertices are added [33].

Feature-based models represent the CAD community’s most recent attempt to capture, or enforce, design practice as well as to facilitate transferring models from the area of design to manufacturing. They restrict the designer to a set of well-defined operations, based on common mechanical features, which are used to describe a part. This set of features strives to be powerful enough to design most manufactured parts but structured enough to ease the design process by limiting the possible topologies to those that are readily machined. Feature-based design is discussed further in Section 2.6.

For more information on model types and their use in object recognition, see Besl and Jain [1], and for surface reconstruction, see Bolle and Vemuri [3].

2.3 Data Segmentation

Besl and Jain describe segmentation as surface characterization. "Surface characterization is the computational process of partitioning surfaces into regions with equivalent characteristics" [1].

The goal of data segmentation is to associate data points with the hypothesized feature they represent. Proper classification of data points into their associated features is necessary before fitting can begin. Least squares fitting supposes a zero mean Gaussian distribution of error. Any points associated with the wrong feature act as outliers, greatly disrupting the fitting process. It is often the case that the points associated with the boundaries between features contain the most noise and thus care needs to be taken to segment out large and/or more easily identifiable features first in an attempt to diminish this problem.

Segmentation methods initially came from the vision community where image partitioning was of key interest. Two primary methods exist for segmentation of a range image: edge based and region based segmentation [24]. Edge based methods use discontinuities to encircle a region that is then considered classified. Region based methods attempt to classify points based on local properties, such as intensity value, orientation, or curvature. All neighboring points that have similar properties are grouped into the same region.

Segmentation of 3D point clouds relies on two approaches. The first and most often seen is bottom-up segmentation, but recent work suggests top-down segmentation as an alternative [36]. In a bottom-up segmentation, subfeatures, such as planes, lines, and arcs, are identified and then combined into shapes such as pockets or outlines. This technique can fail when the data is extremely noisy or sparse, or when the surface does not conform to standard assumptions of smoothness and local uniformity. Attempts to overcome these problems utilize various robust segmentation routines that can tolerate various percentages of outliers.

Owen [24] suggests that a top-down segmentation approach is advantageous. The idea behind top-down segmentation is that if some source (e.g., an interactive user) can identify the top-level feature, such as a pocket, then the computer can produce the low level geometry associated with the feature. The top-level feature provides constraints on the hypothesized model that can be used to help classify a data point as included or excluded.

Various examples of segmentation and fitting techniques are detailed in the next section. Many of these techniques come from the machine vision community where image reasoning necessitates a good classification of the image regions. For further reference see Besl and Jain [1].

2.4 Data Fitting

Data is produced from sensors in an attempt to describe a phenomenon. Often there is some amount of noise in the data because of the inability of the sensor to perfectly capture its subject. It is the object of the data fitting process to produce a model which best describes the sensed object based on this data. Traditional methods employ generic modeling primitives to approximate a wide variety of forms. Newer techniques utilize domain specific models in an effort to overcome the error associated with the sensing process and produce more accurate representations.

General data fitting techniques include functional approximation and interpolation. In the simplest case, interpolation techniques fit functions directly through the measured data points. Approximation techniques fit functions in the neighborhood of the data points, attempting to minimize some error function. Interpolation is often used in the design process where the designer represents a shape, such as the profile of the part, by several points and asks the computer to connect them via primitives such as arcs, lines, or splines. Approximation is used to fit large amounts of usually noisy data. A well-known error criterion is the weighted least-squares function. This function can be described as:

$$E = \frac{1}{r} * \sum_{(r=1)}^{m} (w_r * \text{distance}(p_r, Z))$$

An attempt is made to minimize the error function E, where w is a vector of weights (often uniformly set to one), r is the number of data elements, p is the data vector, and Z is the model. By minimizing the RMS (Root of the Mean of the Squares) error, an optimal fit can be achieved on data that has zero-mean Gaussian noise. Analysis of the sensor used in this work shows it to produce approximately normally distributed data with identifiable areas of systematic error.

For linear equations, a closed form solution can be found using the least squares fitting technique. Because it is not possible to find a general solution to curve estimation, iterative methods are necessary [34]. Iterative methods describe the broad area of algorithms that attempt to find maximums (or minimums) in data spaces via some sort of search or iterative approximation. These numerical routines often assume a continuous functional distribution and follow derivative information in an attempt to descend to the lowest error area. Multivariate functions describing complex CAD modes are often nonconvex, thus having multiple maximums and requiring either good initial guesses, or specialized global optimization techniques.

Many CAD and reverse engineering packages utilize spline based models [4], [10], [19], [30]. Splines supply a mathematically sound representation for 3D curves and surfaces that provide many nice properties such as smoothness constraints and data reduction. Splines can also be broken up into local piece-wise smooth sections to model more complex geometry. These local sections can be modified without affecting any other part of the surface. Unfortunately, splines can actually over fit the data. Because splines are a generic approximation of a surface they can undulate through the noise reducing the error to the data. This can produce a curved surface where a lower DOF surface, such as a line or arc, is more appropriate.

To familiarize the reader with data fitting and segmentation techniques in the area of pattern recognition and image analysis, several methods published in this area are reviewed below.

Han et al. [13] suggest that for industrial parts, it is sufficient to identify a set of features that represent a majority of such parts. These features include planes, cylinders, and spheres. They use an image-based approach, generating normals associated with full regions in the image, separated by planes or other boundaries. From the normals, they predict geometric features. This approach is not appropriate to mechanical parts because of the reliance on inappropriate primitives.

Jain and Naik [19] describe a system for developing spline-based descriptions of objects from range images.[9] Their system is divided into a robust segmentation algorithm and a spline-based fitter. The segmentation algorithm, based on the work by Besl [2], computes HK regions (H is the mean curvature; K is the Gaussian curvature). These HK regions describe various surface properties such as peaks, ridges, valleys, flat areas, etc. The spline-based fitter then applies a least-squares fit to each subregion. The claim is made that this system can produce CAD models regardless of the surface types found in the image.

Hoppe et al. [16] suggest a method of surface fitting based on polygonal meshes. They suggest that surface fitting and function reconstruction are two distinct classes of problems. The idea is to produce a surface that approximates the true surface based on data points on or around the surface. Their method requires no segmentation because the mesh does not exploit any partial structure in the data. They iteratively build up a polygonal mesh based on the assumption that the object can be described as a collection of piece-wise linear surfaces. Although this is a general-purpose technique, it lacks the powerful data reduction abilities of functional approximation and does not classify local areas of data into separate features useful in CAD/CAM applications.

Chen and Medioni [5] also produce polygonal meshes. They insert a balloon (polygonal sphere) into the volume of the object and then inflate it until it contacts the surface, where it becomes locked down. At various stages they increase the size of the balloon so that the polygonal mesh will keep an average polygon size.

Delingette et al. [8] discuss a polygonal segmentation and fitting algorithm that uses feature information to drive the fitting process. They minimize an error criterion based on smoothness energy, feature energy, data energy, kinetic energy, and Raleigh dissipation energy. By combining these forces, they attempt to deform a generic surface (a tessellated icosahedron) into the shape defined by the data points. They present results on sculptured surfaces as well as polyhedral shapes.

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[9] Range images comprise a 2D array of pixels but instead of intensity information, the pixels contain the distance from the camera to the object.

Taubin [34] discusses the problems of parametric curve fitting in two and three dimensions. He suggests that fitting should be based on the mean square distance from data points to the curve or surface, but that this value cannot easily be computed and thus it is necessary to approximate the distance to the curve. He also proposes that generalized eigenvector fits can provide a good initial estimate to iterative techniques. Finally he discusses the idea of interest regions and gives a variable-ordered segmentation algorithm for classifying them.

Yu et al. [40] are concerned with robust segmentation in the face of outliers. They propose a system that can tolerate up to 80% outliers based on measuring residual consensus, using a compressed histogram method. Their method is useful in segmenting out planar and quadratic regions from a range image. The key element is a random selection of data points, a fit to these points, and a comparison with the complete set of random fits. The fit with the most power based on a histogramming scheme is chosen to represent the data. Like many generalized approaches discussed above, neither the accuracy of the approach nor the model generated are suitable for CAD/CAM applications.

2.5 Dimensioning and Tolerance Information

Dimensioning refers to detailing the size of the geometric structures of a part. Tolerancing refers to the process of assigning error ranges with respect to a feature as described by an engineering drawing or a CAD model. Combined, they define the functional limits of a part’s geometry and describe the allowed geometric relations between elements of the part [29]. Both activities are important during the design phase as they pass on information to the manufacturer detailing the required precision necessary for the manufacturing process.

A reverse engineered CAD model must provide dimensions for the part of interest, preferably in a manner consistent with modern design and manufacturing systems. It should also provide tolerance information on the part. This tolerance information should not only represent error associated with the reverse engineering process, but also represent the new set of tolerances associated with machining new parts of this type. Although determining the error associated with the reverse engineering process is conceptually straightforward, estimating original tolerances can be difficult to impossible based only on the sensed data. Such information can often be hypothesized (by a domain expert) based on understanding of the parts functionality.

The field of dimensioning and tolerancing describes the allowable error for a new part in terms of the violation of certain properties of the part, such as the diameter of the holes or the planarity of the pocket walls. This type of information forms a foundation for the constraints used in this dissertation. Current efforts to automate the creation of tolerances from engineering drawings have many of the same problems faced in the reverse engineering community. For example, dimensioning and tolerancing formulations contain a great deal of implicit information that an expert engineer automatically incorporates into the manufacture (or reverse engineering) of a part. An automated process cannot readily extract this information. Likewise, the creation of a new part model can benefit from the explicit information found in the scanned part data as well as the implicit information known about the manufacturing process.

Geometric tolerances describe the usual specifications found on an engineering drawing which detail the relation between features and subfeature. Four tolerance groups have been identified [20]:

1. Form tolerances, controlling the departure of the shape from the true shape. These include: straightness, flatness, roundness, cylindricality, line profile, and surface profile.
2. Attitude tolerances, controlling the rotation relation between features. These include: parallelism, squareness, and angularity.
3. Location tolerances, controlling the translation between features. These include: position, concentricity, and symmetry.
4. Runout tolerances, controlling the amount of wobble when a cylindrical shape is rotated about its axis.

A properly constrained model contains exactly enough dimensioning information to define the part without over or under-constraining it. Current models are constructed using a forward-chaining paradigm, which requires that the part be built in such a way that all the geometric elements are created in a step-by-step manner [20]. Using this insight into the design process allows for a better approach to the reverse engineering problem. These tolerance groups represent the type of information that must be represented in order to accomplish the current work on constraint-based optimization.

2.6 Feature-Based Design

Recently, a push has been seen to develop feature-based tools for modeling and reverse engineering. The term feature in CAD/CAM has come to represent an encapsulation of data representing both the form and the function of the object. Features represent many properties associated with the design and production of a part. Design features provide information associated with the designer's intent. Manufacturing features provide insight into the manufacturing process for the object. The distinctions and transformations between design and manufacturing features are the subject of continuing research [6]. The term form feature is used to represent the stored geometrical data. Form features can be represented as volumetric features, referring to the solid volume of the feature, or as surface features, designating the surfaces exposed when adding or subtracting a volumetric model from a part. In addition, features can be used to represent design intent via assembly features (how parts interact), material features (what parts are made of), and precision features (tolerance specifications).

The use of a feature-based paradigm is important for two reasons: First, features form a natural and efficient model for fitting data; second, many parts are designed either directly or intuitively along feature-based lines using standard design practices, and thus features usually correspond to the designer's intent and accurately represent the data acquired from real parts.

Work by Shah [32], Shah and Rogers [31], and Cunningham and Dixon [6], form a foundation for the use of feature-based design and modeling in manufacturing. A designer will choose a feature, such as a hole or pocket, and sketch (or otherwise define) the geometry [33]. The CAD system implements any implicit properties that hold on the geometry of the feature, such as smooth transitions between arcs and lines. Further, the designer can specify tolerance information as well as other design intent and save this with the model.

The need exists for translating design features into manufacturing features that can be directly incorporated into NC machining code. Cunningham [6] states that all design features should be formed in such a manner as to be translatable into manufacturing features that can be directly machined. Some features are common to both manufacturing and design, such as holes, while others require specific translation. The features used in this work are common to both design and manufacture.

Merat [21] uses a set of design features for inspection planning and associates inspection hints with each feature. In a similar manner, the methods utilized in this dissertation apply constraints on the primitives used for features of a part, as well as across sets of features, bringing knowledge to bear on the model recreation process.

Feature recognition is often necessary for producing a machining plan from a low level geometric model of a part. This is similar to reverse engineering, but without the problem of noise. An extensive review of this process can be found in Shah [32] and Vandenbrande [38]. Shah additionally subdivides the area into machining-region recognition (the development of tool paths) and that of actual feature recognition, arguably the more complex problem. In both cases, the idea is to proceed from a geometric description of the part, such as a B-Rep (or in limited cases a CSG model), and derive actual features or milling volumes. Both researchers utilize bottom-up segmentation of the geometric model, thus attempting to locate definable low-level patterns in the data that can be combined and built up into fully defined features. This amounts to a combinatorial search problem, with heuristics to direct the search.

Vandenbrande [38] shows some of the obstacles associated with a bottom-up approach, pointing out the difficulties of identifying features that interact and thus obscure each other. The alternative is based on a top-down segmentation of the data. This is accomplished by allowing a human operator to interactively point out high-level features. Thompson et al. [36] have proposed that manufactured features can more easily and justifiably be located and recovered by recognizing the power and utility of an interactive process utilizing an expert human. The expert identifies the overall feature, such as a pocket, and the computer builds the underlying geometry of the feature. This approach can save time and deal with interacting features. Further, the user can evaluate the recreation and modify it as necessary.

The work of [6], [10], [14], [33], and others provides confidence that manufactured parts are designed along common practices. Feature-based CAD systems are a natural out growth of practices common to engineering design [10]. This idea provides the key to Owen's Masters thesis [24]. Owen's work advances the following the premises:

- Features form a natural parametric foundation for accurately fitting and representing data in the reverse engineering process when applied to mechanical parts.
- Key features represent extruded profiles that can successfully be fit in 2D.
- An interactive system utilizing a knowledgeable user can accurately and efficiently segment data and verify fits, using a top-down approach.

2.7 Geometric Constraints

Fundamentally, this dissertation is about taking physical representations of geometry and producing a mathematical model describing the physical object. Two key issues are: 1) how to represent the geometry, and 2) how to impose constraints on the fitting process to construct geometry that behaves in a manner true to the physical world. The following section considers current research in the area of representing and manipulating geometry in the design of manufactured objects.

Originally, the entire weight of making sure a engineering drawing was consistent fell on the designer. CAD systems were developed to put this onus on the computer. Many strategies have been tried, beginning as early as the 1960s with Sutherland's Sketchpad program. Hsu's dissertation [17] attempts to address the idea of geometric constraint solving in the design process. He lists several criteria for an ideal constraint solver:

1. Reliability - Derive all possible solutions (if required).
2. Predictability - Do not jump erratically through the solution space and should provide a way for a human to control the results.
3. Efficiency - Allow interactive response times.
4. Robustness - Handle over and under-constrained problems.
5. Generality - Handle a wide variety of constraint types and not be restricted to any specific dimensions.

Couching these requirements in terms of a computer aided reverse engineering system gives:

1. Reliability - The algorithm should derive a model that is consistent with the data given it and its knowledge of the design and manufacturing processes.
2. Predictability - The algorithm should come up with the simplest accurate solution. An interactive user should be able to guide the process.
3. Efficiency - The algorithm should run at interactive speeds.
4. Robustness - The algorithm should be able to handle over and under-constrained hypothesized features.
5. Generality - The algorithm should be able to handle a wide variety of parts and inter-related constraints and not be restricted to any specific dimensions.

Given these criteria, Hsu defines four methods developed to address the constraint-solving problem. These include propagation methods, numerical methods, constructive methods, and algebraic methods.

Constraint propagation is the process of representing the geometrical constraints in the form of an acyclic graph. The graph contains nodes representing the variables or constants defining the geometry and the edges of the graph represent the relationships between the geometry. Once the acyclic graph is built, values are propagated throughout until a solution is found. The graph cannot contain cycles of dependencies in order to ensure a solution. To overcome this weakness, this method must be combined with numerical approaches.

Numerical methods have been briefly described previously in relation to data fitting. In these cases the geometry is represented as algebraic formulas and constraints are created by relating variables across equations. Once a global equation is developed describing the geometry of the part, an iterative process is invoked to find a minimum error fit. Unfortunately, these techniques are sometimes unpredictable and can have difficulties converging.

A newer method of constraint solving which applies to problems solvable by ruler-and-compass construction is known as the constructive approach. Constructive methods are an extension to standard propagation techniques, differing in the way constraints are ordered for evaluation. Hsu describes two approaches: rule-based and graph-based. In the rule-based approach, geometric constraints are represented symbolically. Rewrite rules are utilized to simplify geometry and reduce DOFs. Unfortunately, rule-based systems tend to be slow. Graph-based approaches consist of two steps. One, a top-down phase is entered where the graph is analyzed and a sequence of constructive steps is derived. Two, a bottom-up phase occurs where the construction steps are carried out and the model is constructed.

The final group of constraint solving techniques uses algebraic methods. The geometric constraints are written as algebraic formulas, which are then combined and reduced using elimination methods. Algebraic methods tend to be extremely slow and often have exponential complexity. Table 2-1 is taken from [17] and summarizes the behaviors of the various methods.

2.8 Sensors and Scanning

The current generation of sensors provides three main operational groups (hand held, manually controlled, and automatic) and consists of two separate methods (touch sensing and noncontact sensing). Hand held and manually driven sensors include traditional sensors such as calipers and micrometers, as well as modern coordinate measuring machines. These devices fall within the paradigm of touch sensing, requiring some sort of probe to physically contact the surface of the part. Non-contact sensors,

Table 2-1: Method Analysis

	Reliability	Predictability	Efficiency	Robustness	Generality
Propagation	Yes	Yes	Fast	Yes	No
Newton’s	No	No	Moderate	No	Yes
Homotopy	Yes	?	Slow	No	Yes
Rule-based	Yes	Yes	Slow	Yes	No
Graph-based	Yes	Yes	Fast	Yes	No
Algebraic	Yes	?	Very slow	Yes	Yes

which are usually controlled automatically, include a gamut of devices that depend on light sensing and triangulation to produce 3D surface data.

Hand held sensors, such as micrometers and calipers, are in theory accurate to +/- 2 microns[10] but are subject to human error, and are limited to a small subset of the possible geometrical measurements that can be applied to an object. Their strength lies in measuring static fundamental qualities generic to most parts. Such qualities include hole diameters and part thickness. They can also be specialized to a particular job, such as determining the diameter of a rounded edge.

Touch sensors do not provide a good means for measuring changing contours or other areas, on a part, which are custom designed. For example, to determine the dimensions and geometry of a moderately complex interior pocket contour would require hundreds of accurate touches. To even contemplate this task, one would require some sort of semi-automatic coordinate measuring machine (CMM). Experience with CMMs has shown that, while extremely accurate, they are expensive, hard to utilize, user intensive, and slow. Further, CMMs require customized programming (or manual operation) for every new part upon which they are used. These problems suggest the use of noncontact sensing, cameras and/or lasers, as well as active sensing methods that attempt to remove human interaction from the sensing process.

Noncontact sensors predominately use light to sense the shape of an object. Stereo camera systems triangulate common points on the object base on corresponding points in each view. Laser systems either use time of flight to measure the distance to the object or project the laser spot on to the object and triangulate the distance. Such triangulation systems are the most accurate claiming results in the range of +/-50 microns. These automatic range sensors have an advantage of being fast and requiring minimal user time. Under ideal circumstances, the error associated with the acquired data is evenly distributed and can be averaged out; however, numerous situations encountered while scanning parts produce data points that are less well behaved.

[10] On average repeatable measurements are to +/- 25 microns.

For this work a Digibot II laser scanner was employed (Figure 2-1). This scanner is a fully automatic laser range scanner. The laser projector is translated in the X and Z directions. The part is placed on the rotary table capable of rotating the part 360°. The laser is set at a particular height (in Z) and the part is rotated. A small red laser dot is projected onto the surface of the part, representing a single data point. As the laser strikes the part, two separate light sensing diodes (fixed at 30° offsets from the projection vector) mechanically translate along the X direction sensing for the greatest light reflectance. Given the X offset from the projection spot, a Y value is triangulated. Once a value is calculated, the part is rotated a fixed number of degrees and a new Y value is calculated. This produces a scan contour (or several contours), consisting of consecutive

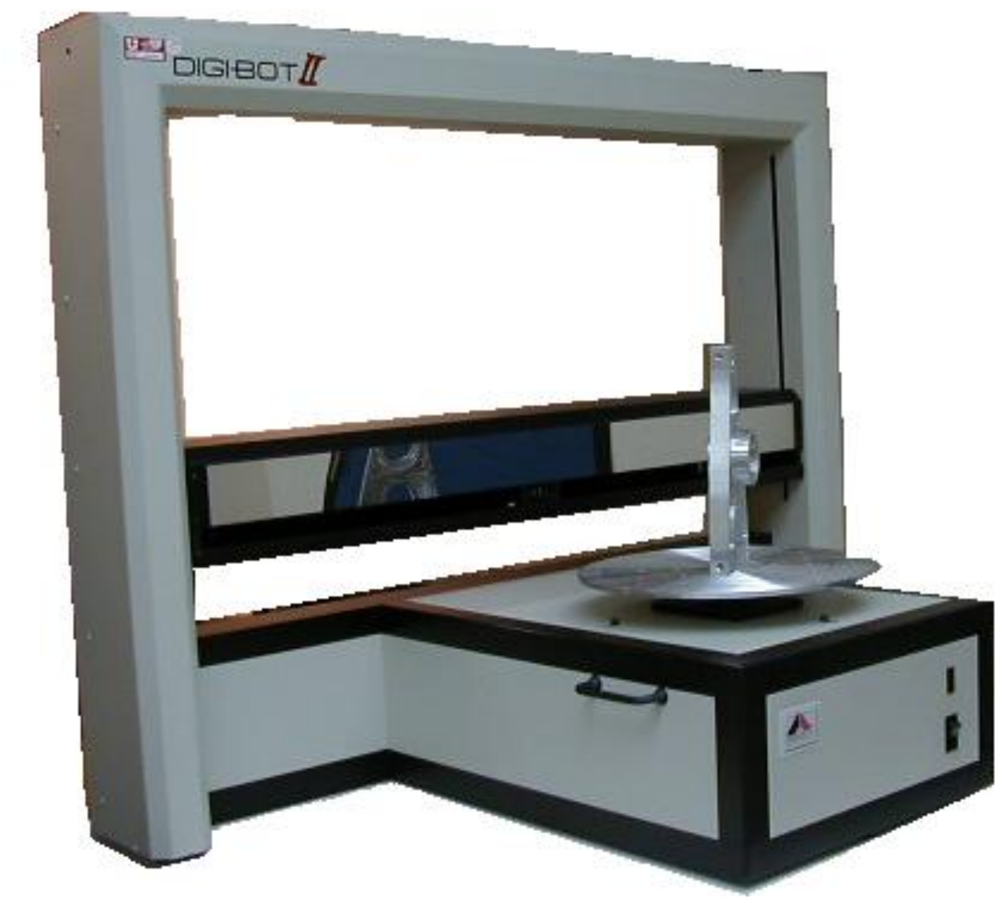


Figure 2-1: Digibot II Laser Scanner

2D points, describing the 2D geometry at a particular Z level. Once these connected contours are completely sensed (to the ability of the machine), the laser projector is raised by a preset amount in the Z direction and a new set of contours is created. This process continues until contours have been created from the bottom of the object to the top. This results in an unorganized (the contour relationship is not used) set of 3D points.

Empirical testing of the Digibot II shows data errors in the range of +/-125 microns and systematic errors ranging up to +/-250 microns or more.[11] The sensing process contains many sources of error. It is not the purpose of this dissertation to address these issues, but instead to take the data "as is" from the sensor and attempt to overcome the noise through the constrained optimization process.

The following problem areas have been identified in association with the Digibot scanner:

- As the surface plane of the part relative to the path of the laser beam approaches 90°, the laser does not accurately reflect back from the object to the sensors. This often requires multiple scans of the part and results in subpar accuracy on sloped surfaces.
- Deep concavities cause the scanner to be unable to sense the location of the laser mark, thus leaving unsensed areas in the data cloud.
- Narrow pockets cause a spreading of the reflectance pattern of the laser. This causes the bottom of pockets to appear to slope inward.
- The surfaces of the scanned object must be able to reflect the laser in a uniform manner. Many machined parts have highly specular metallic surfaces. To scan them requires that they are sprayed with a coating of white powder or paint. The paint thickness averages approximately 25.4 microns in thickness, but varies randomly across the entire model. For high accuracy models, the thickness of the paint must then be analyzed and compensated for.

[11] As a point of reference, NC machining systems can produce parts to the accuracy of +/-25 microns or better. For general purpose parts, accuracy in the range of +/-50 microns is common.

It is apparent that data from even the best scanners are far from perfect. New methods of fitting are necessary to compensate for this lack of accurate data. The use of features and constraints attacks the problem at the fitting stage, not at the sensing stage.

2.9 Optimization

This section reviews the basics of optimization principles and techniques. It is not intended as a complete introduction to numerical optimization, but as a refresher of general principles. For more information refer to [27].

Optimization is the attempt to find the best (or at least a good) solution to a problem where multiple variables are in competition. This is achieved by minimizing the value associated with an error function. In the case of model fitting, the standard function to be minimized is the distance from the empirical data to the surface of a hypothesized model. The model is represented parametrically by a list of values that must be instantiated in order to specify the physical geometry of the object. As these values are modified, the geometry of the model changes and the amount of error between the instantiated object and the data fluctuates.

Given only two variables, the error can be graphed as a topographic map with the Z dimension representing the amount of error caused by various values of X and Y. In higher dimensions, it becomes quickly impossible to visualize the optimization surface. The only certainty is that as the number of dimensions or DOFss increase, the problem becomes much harder to solve. By choosing an appropriate representational form, it is possible to limit the DOFs necessary to fully represent a part's geometry. This both increases the resistance to noise in the data and makes the fitting process more likely to converge to the desired result.

Typical reverse engineering programs tend to localize the optimization to one feature or even one subfeature, thus reducing the DOFs to a reasonable number. When trying to optimize an entire model, even one of only moderate complexity, it is possible to deal with 50-100 DOFs, a situation in which even very fast machines take excessive amounts of time and most algorithms begin to break down. Finally it should be noted that it is seldom the case that the best result is achieved by simply fitting the data. When possible, appropriate models and constraints should applied to the optimization process in order to guide and simplify it.

2.9.1 Numerical Methods

Classical numerical optimization is the attempt to find the maximum or minimum[12] of an equation, or system of equations. For functions of one variable, the goal is to find the value $\mathbf{x}^*$ such that $f(\mathbf{x}) \geq f(\mathbf{x}^*)$ for all $\mathbf{x}$. The function, $f(\mathbf{x})$, is termed the objective function. An example can be seen in Figure 2-2. For this simple function, visual inspection determines $\mathbf{x} = 1$ as a local minimum and $\mathbf{x}$ ~= -0.64 as the global minimum of the function, and no unbounded maximum exists.

Unfortunately, visual inspection is seldom available when solving complex equations, and even an expert's ability to reason about or visualize spaces drops off quickly after more than 3 DOFs. To address this problem, optimization theory has created a class of iterative algorithms which attempt to step through the function starting at an initial value, $\mathbf{x}^{\mathbf{0}}$, evaluating successive values of $\mathbf{x}^{\mathbf{k}}$ until a minimum value, $f(\mathbf{x}^*)$, is found. For multiple dimensions, $\mathbf{x}$ represents a vector of parameter values. Several issues must be addressed to turn this general outline into a usable algorithm:

1. How to choose the initial value, $\mathbf{x}^{\mathbf{0}}$?
2. How to choose the next value, $\mathbf{x}^{\mathbf{k+1}}$?
3. When is a minimum found?
4. Is the minimum a local or global minimum?
5. Are there any constraints that bound the values of $\mathbf{x}$?

Question 1 involves where to start the algorithm. In general, this depends on the character of the function being optimized. The best-behaved functions are those that are convex (or concave). A function is said to be convex if it has the property:

$$f(px + qx') \leq pf(x) + qf(x')$$

[12] This discussion will be in terms of minimums. For maximums simply solve for $-f(\mathrm{x})$.

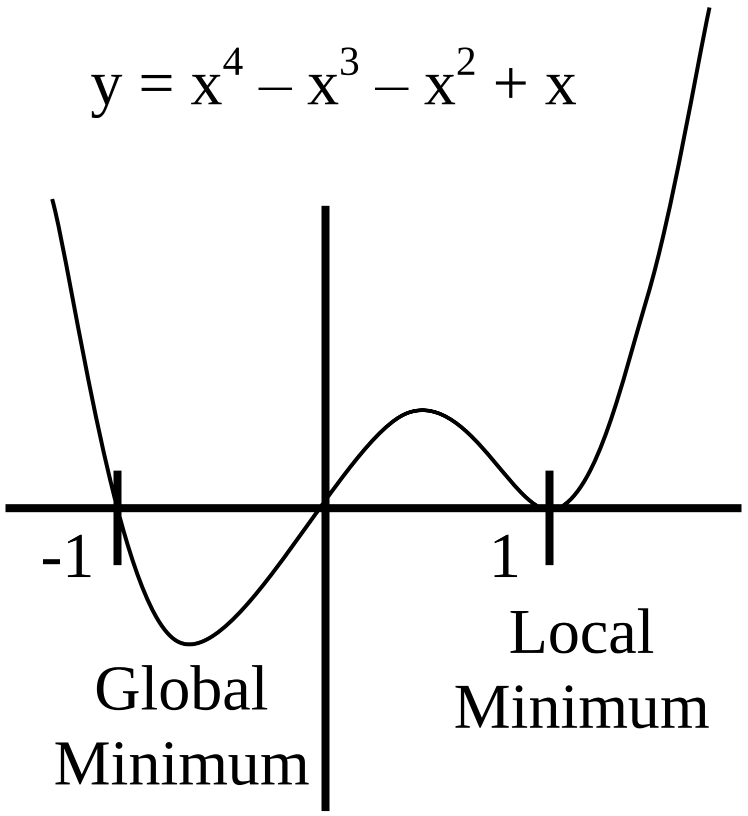


Figure 2-2: Critical Points

for any x, x' in C, and any pair of non-negative scalars p and q. This implies that all points that lie on a line between two points of the function are also in the function (which is certainly not the case for large noisy reverse engineering spaces). It can be shown that convex functions have only one minimum, and therefore that minimum is the global minimum. Similar to convex functions are unimodal functions, in which there is a path of ever decreasing values from any starting point to the global minimum.

Other good characteristics include continuity and differentiability. Derivatives provide information about the local behavior of the function. Further, if the function can be stated analytically, direct reasoning can often be made on it. Unfortunately, many of the functions of interest in the real world are not so well behaved. They often have multiple minima and maxima, have no analytic form, and at best can be differentiated by finite difference techniques. In these cases, the importance of a good starting "guess" cannot be overemphasized. Arbitrary starting points cannot be guaranteed to converge to a global minimum and, at best, will take many more iterations to do so.

Typical initial guesses attempt to start the optimization process as close to a solution as can be determined by some preprocessing. Thus initial guesses are usually made with some "knowledge" of the function. In the case of reverse engineering, each low level geometry element is fit separately and the results are used to start the global optimization process.

Question 2 forms the crux of the iterative process. "How is x(k+1) chosen from x(k)?" The methods under consideration are local descent methods, which attempt to follow the function downhill until a minimum is reached. Thus:

$$x^{(k+1)} = a^{(k)} p^{(k)} * x(k) : f(x^{(k)} > f(x^{(k+1)})$$

where p is a direction vector and a is a step size. The down hill direction can be chosen based on derivative information, giving the equation:

$$0 \geq g(x)^{T} \bullet p$$

where g(x) is the gradient of the function at x. The choice of a is deceptively complex, as too large an a leads to overshooting the proper value and too small an a leads to slow convergence rates or non convergence.

Question 3 concerns when a minimum has been found. A minimum point is a point such that:

$$f(x+\varepsilon) \geq f(x) \forall \varepsilon \mid \varepsilon < \delta .$$

This formula is similar to the definition of the first derivative of a function. This leads to the idea that a necessary condition for a minimum is that the first derivative equals zero. Furthermore, it should be noted that while necessary, this condition is not sufficient to guarantee a minimum, as inflection points also have first derivatives equal to zero.

If the initial guess is close enough to the actual answer, then finding the minimum can be equated to finding the nearest zero crossing of the derivative. Newton's method is one method that can compute zero crossings. This method forms the prototype for many iterative optimization algorithms and will be discussed in greater detail in the next section.

Question 4 ponders the quality of the solution. Is the current minimum a local or global minimum, and if a local minimum, is it good enough? As noted previously, if the function is convex, a strong statement can be made about having found a global minimum. Unfortunately, these are the only types of functions that have been studied sufficiently to make such a claim. Certain optimization algorithms are designated as Global Optimizers[13], and these routines attempt to find the global optimum, usually through techniques such as stochastic search, grid search, or simulated annealing. They have little application to the problem at hand, as the complexity of the space makes them ineffective and a good initial guess is available.

Question 5 introduces the idea of constrained optimization. Two techniques are combined in the reverse engineering application. The first method is known as an interior-point method; it constrains all iterations of the solution to remain in feasible regions (meaning all constraints are satisfied.) For the purposes of this work, this is achieved via a model complexity reduction based on the constraints. The details are presented in chapter four. The second method is an exterior-point method; this method allows the intermediate parameter values to violate certain of the constraints in an effort to escape local minima. This is achieved using penalty functions (which are discussed in Section 2.9.4).

By answering these five questions in a manner consistent with the reverse engineering process, it is possible to utilize optimization for the purpose of improved model creation.

[13] The term “Global Optimizer” is taken from the optimization literature. Future references to the term “global optimization” in this dissertation refer to the process of optimizing all surfaces of a model at one time.

2.9.2 Newton's Method

Newton's method has two properties of interest: 1) it is an iterative algorithm which improves with each iteration,[14] and is thus a prototype of many optimization routines. 2) It solves zero crossing problems, which are useful in optimization. Newton's method is traditionally described as a means for solving first derivatives of an equation, but to understand the mechanics of the method, the problem of solving one equation in one unknown is presented.

Consider finding the square root of 3. This equation can be formulated as finding the root (or zero crossing) of:

$$x^2 - 3 = 0$$

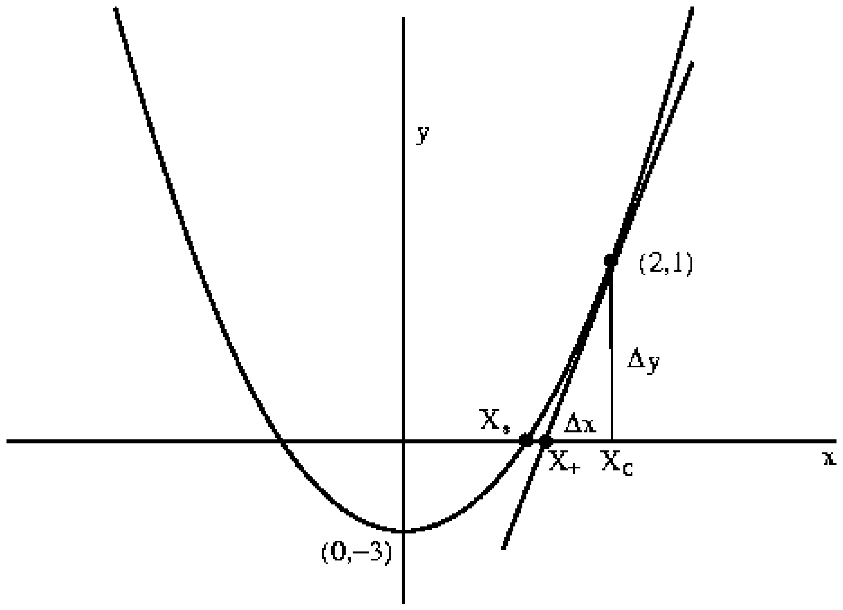
Starting with an initial guess, say 3.0, Newton's method attempts to calculate progressively better values of x based on intersecting the tangent line at the current value, x_c, with the x-axis (see Figure 2-3).

Thus, at each step, Newton's method computes an approximate local model (in this case, a line) of the equation and solves for it. As long as the starting guess is close to the answer and the derivative has a nonzero lower bound, then Newton's method converges quadratically to the desired result (each iteration increases the number of correct decimal places by a factor of two).

Newton's method has a few notable characteristics:

1. It relies on being able to take the derivative of the function.
2. It is not globally convergent in all cases (perhaps most cases).
3. If the initial value is close to a local minimum or maximum (hence the first derivative is approximately zero), then the next approximation may be singular or ill conditioned.
4. Certain pathological cases can cause the iterations to repeat. Consider taking the Newton value of arc tangent (x) at x ~= 1.39175

[14] Newton's method will improve, or remain constant, as long as the current guess is within a close range to the answer being sought.



$$x_+ = x_c - \frac{f(x_c)}{f'(x_c)}$$

Iteration	Value
1	3
2	2
3	1.75
4	1.7321428
5	1.7320508

Figure 2-3: Newton's Method

Despite these shortcomings Newton's method furnishes a powerful tool for the use in and prototype for more advanced optimization methods.

2.9.3 Conjugate Direction Methods

In high dimensional spaces, it is not visually or intuitively apparent what direction to take to proceed toward the minimum of the function. A simple approach is to fix all the variables except one, modify that one variable until a local minimum (in one dimension) is found. Now fix that variable and modify another variable. Continue until no modification of any variable improves the function evaluation.

This search is known as the univariate search method. It only requires function evaluations and 1D optimizations. For some problems, it is an effective approach. Unfortunately, there are many problems where this is not the case. Consider Figure 2-4: for (a) there is little interaction between the variables and the search converges quickly; for (b) the search breaks down into many very small zigzags toward the goal.

Ideally, a direction would be chosen that attempts to move directly toward the minimum, rather than only in one dimension. The univariate search attempts to optimize along the one-dimensional vectors (1,0) and (0,1). A small logical leap suggests the use of other directional vectors. In (b), the vector (-1,1) forms a much better optimization path from the starting point. The question then becomes, what other directions to search in. If the search directions are too closely aligned, convergence becomes difficult.

Powell [26] suggests that the only direction vectors to be utilized should be orthogonal, or conjugate, to each other. Further Powell provides an algorithm for determining these directions as follows: Start a univariate search. After one iteration along all axes, replace one of the univariate directions with the vector formed by subtracting the start position from the current position. Move along this direction until a

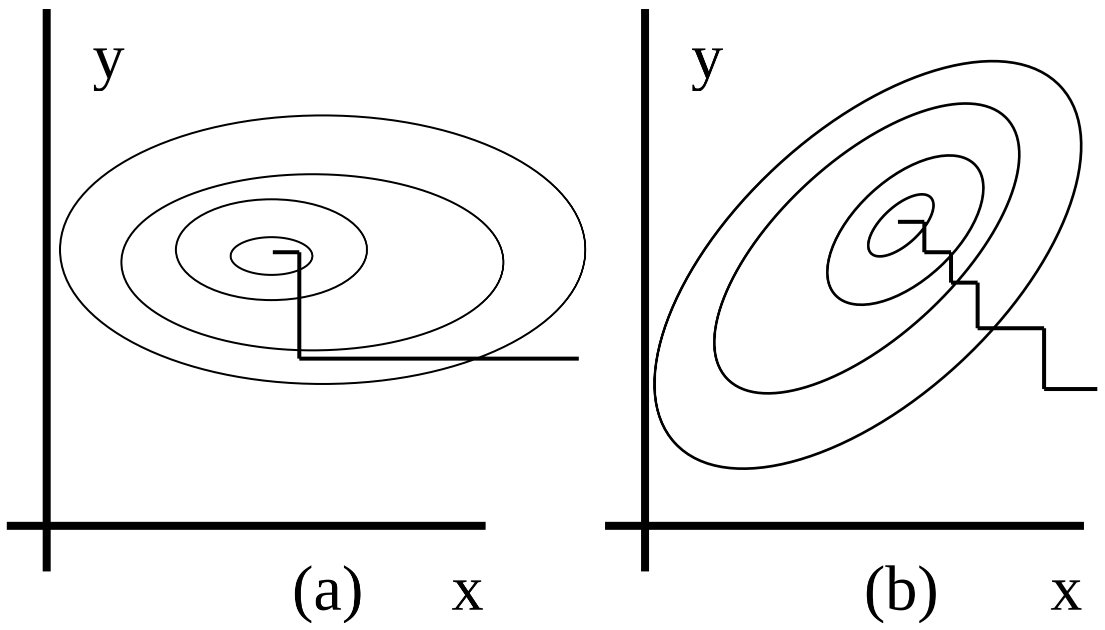


Figure 2-4: Univariate Search

minimum is found and then repeat, continuing to replace the initial univariate directions with the new vectors. After an additional iteration for each dimension, all the directional vectors will be conjugate.

Both Newton's method and Powell's method use only function evaluations. If gradient information can be derived efficiently and accurately, then it should be used. The gradient is the direction of greatest increase of the function and can be calculated based on first derivatives. Using gradients results in a steepest descent algorithm.

At any point in the optimization, a new direction is chosen based on the gradient at the current point and a 1D optimization along that vector. Steepest descent algorithms attempt to improve the local optimization of a function to the greatest degree with each step.

Often it is the case that gradients are not analytically available. In this case, finite differences can be used to approximate the Jacobian.

$$J \approx (g_c)_i = \frac{f(x_c + h_i) - f(x_c)}{h_i}, i = 1, \ldots, n$$

As h_i approaches 0, $(g_c)_i$ approaches the gradient. Finite difference routines must choose a value of h_c sufficiently small to approach the gradient without falling subject to noise in the data.

2.9.4 Boundary and Penalty Methods

There is great interest in solving the constrained optimization problem using the techniques already established for unconstrained optimization. This transformation is usually effected by building a new function, T, as a combination of the original error function, F, and a boundary function, Φ, which defines how far the constraints have been violated:

$$T(x, r) = F(x) + \Phi(c(x),r)$$

The two best-known boundary methods, Φ, are barrier and penalty methods. Barrier methods form steep walls around the feasible values of x. Thus $\Phi(c(x),r)$ would equal zero when the constraints are met, but approach infinity as the constraints are violated. Barrier methods are interior point algorithms, requiring the initial and all subsequent iterations to be feasible points.

The second form of boundary methods uses what are termed penalty functions. Rather than forming walls around the feasible area, slopes are used which allow the constraints to be mildly violated. This allows the initial points to be outside the feasible realm. As the iterations increase, the penalty becomes higher, forcing the optimization into the feasible region.

Both methods suffer from increasingly steep-sided valley effects as the controlling parameter is intensified.

2.9.5 Simplex Search Method

The simplex[15] search [23] is a direct-search method that relies only on function evaluations to compute the minimum. A simplex is an N + 1 dimensional structure that attempts to flow downhill until a minimum is found. In the case of a 2D space, the simplex is a triangle. Starting with one axis of the simplex, vectors to the other points form a basis for the space. Each point on the simplex is evaluated and then several operations can take place in an attempt to move the simplex closer to a local minimum:

- Reflection - the largest valued point is reflected through the hyper-plane formed by the other points.
- Reflection and expansion - a new point is chosen along the line from the highest point through the hyper-plane. This point is pushed as far as possible from the hyper-plane in order to make larger amounts of progress in the downhill direction.
- Contraction (multiple contractions) - when the simplex encounters the bottom of a valley or tries to move through a narrow gap in the function space, the reflection step will often fail because the reflected point is not making new progress toward a minimum. Therefore, the size of the simplex is reduced in one or more

[15] Not related to the Linear Programming Simplex routine.

directions in an attempt to ooze through the area (thus attempting to make the scale of the simplex more appropriate to the scale of the function in the current area of concern.)

The simplex routine does not use derivative information and is frugal with the number of function evaluations. These functions can be implemented as codes to create the model based on the new parameterizations and to compute the error from the new model to the data set. For these reasons it has proved an effective general-purpose optimization routine. See [23] for more information including an implementation of the simplex algorithm.

2.9.6 Constrained Optimization

A more rigorous description of the optimization process involves the concept of an objective function. This function, F(x), provides a measure of the current success of the optimization based on the instantiation of all appropriate variables (the parameter vector x). The goal of the optimization process is to find a parameter vector x which minimizes the value of F(x). Without applying any constraints, this process is known as the unconstrained optimization process.

Constrained optimization attempts to minimize F(x) while satisfying the constraint functions G(x) and H(x), thus:

Minimize F(x)

subject to: G(x) = 0 for all i

H(x) > = 0 for all j

Once F, G, and H have been appropriate formulated, generic optimization software can be used to attempt to find a minimum. Many optimization routines exist. This research uses both the simplex search routine and Powell's method.

For model building, the parameter vector x is used to instantiate a model representing the object. For the purposes of this dissertation, the topology of the object is determined and confirmed prior to the optimization process. Once x is generated, the static list of data points is compared against the hypothesized model using a root mean squares error criterion.

CHAPTER 3

DESIGN ARTIFACTS, CONSTRAINTS, AND REVERSE ENGINEERING

Manmade objects often embody certain properties, in terms of geometric shape, that are explicitly and implicitly laid out during their design. These properties, or design artifacts, come about because they fulfill certain desirable traits, such as simplicity, effectiveness, functionality, and manufacturability. An understanding of an object's properties can be used to increase the effectiveness of identifying, classifying, or modeling a part. This is especially true for the design and manufacturing of 2.5D mechanical parts. By identifying components of the design and manufacturing process and enforcing them via mathematical constraints during model reconstruction, more realistic and accurate models can be produced.

Consider the genre of 2.5D manufactured parts. Although these parts vary largely in function and size, they have many properties in common. An example would be the use of planar surfaces that ensure two objects fit flush against each other. A noisy data set taken from a planar surface may well be more accurately approximated, in terms of distance from the data to the model, by a sculptured surface, yet domain specific knowledge refutes the likelihood of such a representation because designers purposefully choose planar surfaces for their functional efficiency, simplicity, and machining ease. Thus a plane fit to the data would exhibit more error in relation to the data than a free-form surface, but would be a better approximation to the design intent behind the object. This is the fundamental aspect of how knowledge can benefit the model recreation process.

3.1 Reverse Engineering an Exemplar Part

Consider Figure 3-1 depicting the Lower Link, a 2.5D part used as in the suspension assembly for a Mini-Baja racer. The goal of the reverse engineering process is not only to create a model that accurately represents this shape, but more importantly, recreates the geometry in such a manner as to reproduce a model that reflects what the original designer intended. Such a model can then be used to produce replacement parts or to make modifications to the original part's design.

An engineer has two sets of information to work with when recreating a model:

1. Physical measurements of the object.
2. Design and manufacturing knowledge about the object.

All model reconstruction techniques utilize some form of sensed data, such as range images or point clouds as the primary source of information. The difference lies in how the data is transformed into geometry. Most methods fit generic forms such as triangulated meshes, B-Reps, or generalized cones (or other geometric primitives) directly to the data.

A discerning look at the Lower Link and an understanding of the uses and

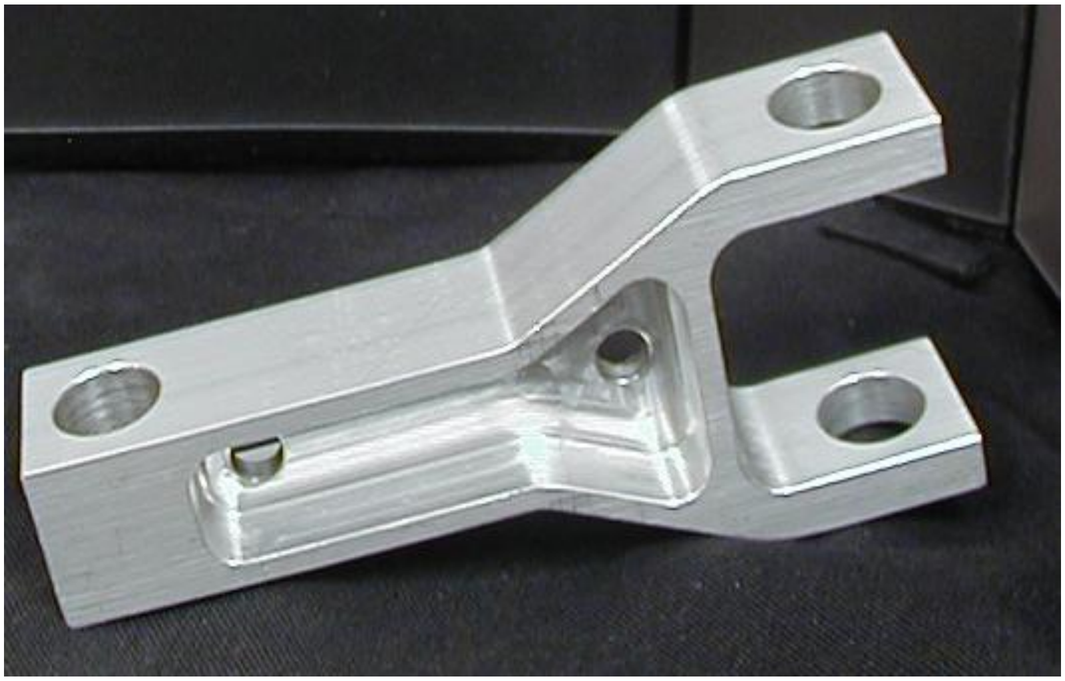


Figure 3-1: Lower Link

tolerances necessary for a valid manufacturable model belies the applicability of these general forms and suggests that a structured, domain specific method is required.

Figure 3-2 represents the data cloud constructed by sensing the Lower Link. Notice the general noise, the rounding of corners, and the systematic error in the pockets (as seen by the sloped line in the third view). The average sensing error along the exterior profile of the part (where the best data is collected) is approximately 130 microns. The worst case data points were over 900 microns off the surface. For data on the interior pockets the average error is 300 microns and worst case is 1675 microns. General fitting techniques cannot adequately compensate for these inaccuracies if the resulting model is to be manufactured as a valid replacement for the original.

Algorithms exist that will automatically transform a data cloud into an irregular triangulated mesh (Figure 3-3) forming a model that can be machined. This "fitting" process can be applied directly to the data, and thus no knowledge of the object's function is needed. Although the reconstructed mesh closely conforms to the data cloud, there is no guarantee of topological equivalence with the original design; more importantly, while each face of the mesh, on average, may be a good local approximation to the surface of the original model, the combined elements of the mesh unduly fluctuate based on local data inaccuracies, and thus form a poor representation of the true machined surfaces of the object.

From this evidence it is asserted that knowledge about the design and manufacture of a part is as important as the sensed data, which should be considered only one of several associated sources of information used to generate a model. By analyzing hypothesized geometry both in relation to the sensed data and based on knowledge of how these types of parts are designed, manufactured, and assembled, a higher quality model can be constructed. In Chapter 4, a focus is made on how this knowledge can be formalized for use with general optimization techniques to fit both the data and the functionality of the desired part simultaneously.

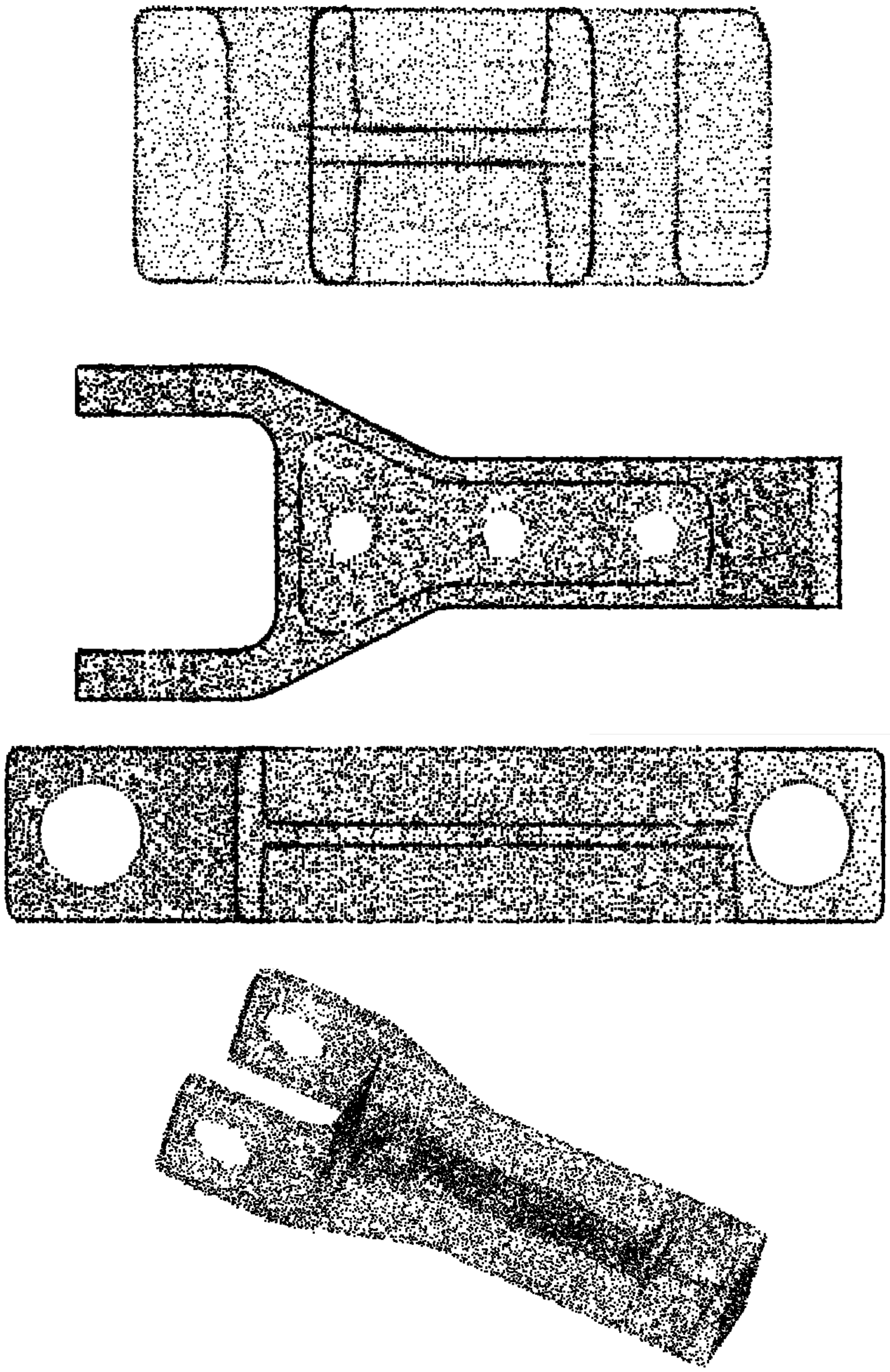


Figure 3-2: Views of the Lower Link Data Cloud

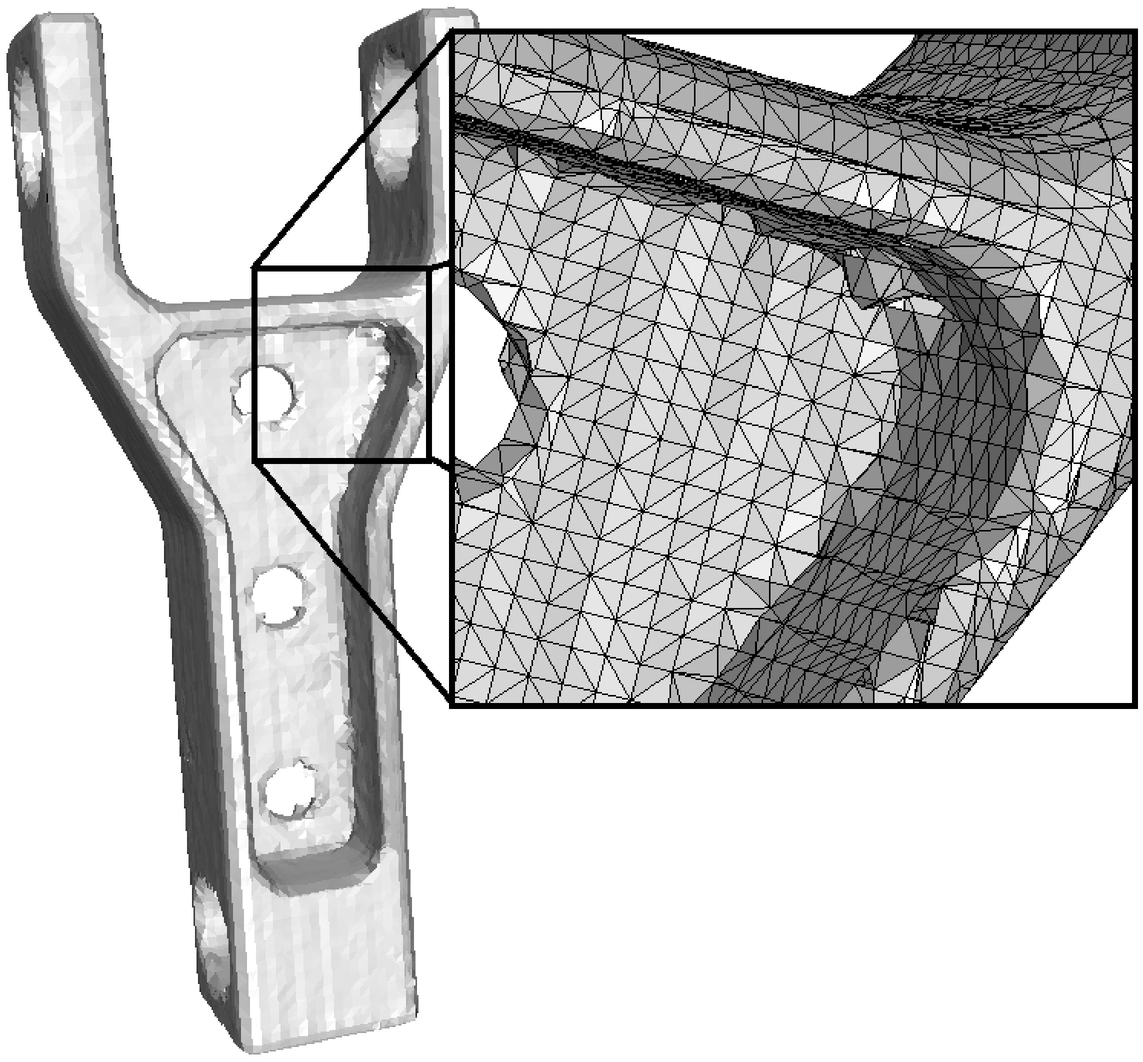


Figure 3-3: Triangulated Mesh Representation (Lower Link)

3.1.1 Local Constraints

Knowledge of the design process indicates that most, if not all, geometric elements of a part are designed from simple well-defined shapes that serve specific purposes. Expert engineers use this information to simplify the job of modeling a part. By quantifying the possible design artifacts, an algorithmic fitting process can also be greatly simplified, resulting in a more accurate reconstruction.

By applying knowledge of 2.5D manufacturing and design techniques, several educated assumptions about the general geometry of the Lower Link can be made (i.e., the main "flat" surfaces are presumed to be planar, the curves appear to be continuous, there are holes, profiles, and pockets in the part, etc.). By exploiting the fact that arbitrary geometry is unlikely, more appropriate models can be developed for reverse engineering. Owen [24] suggests a feature-based breakdown of 2.5D parts. Features encapsulate the form and function of a part's geometry using a mathematical formalism that constrains the possible configurations of a part. These well-defined geometric primitives reduce the possible DOFs of the constructed model and force the geometry to match a hypothesized version of the designed model. For example, they enforce a smoothness quality[16] between the arcs and line segments in the contours.

One of the important aspects about 2.5D features is that they represent a piece of 2D geometry that has been extruded into the third dimension. This realization allows a two-tiered fitting process, beginning by extracting and fitting the bounding planes, and then followed by a 2D fit of the profile curve data. To recreate the original surfaces, these curves are then extruded normal to the bounding planes.[17]

Figure 3-4 depicts a feature-based decomposition of the Lower Link. The features of interest include holes, profiles, and pockets (all contained within their bounding planes). During the reverse engineering session, the engineer would identify each contour and hole for segmentation and fitting.

Features inherently enforce certain low level geometric constraints. In design this limits the set of constructed entities to those that are easily manufactured. In reverse engineering, this has the affect of forcing the fitting process to produce geometry that is likely to be seen in real world parts.

3.1.2 Global Constraints

The application of domain specific features is not in itself a panacea, as it is not powerful l enough to fully exploit and capture all the functional properties of an object.

[16] Specifically C1 continuity.

[17] In practice the planes are fit to the data and then the segmented 3D data representing the feature are projected onto one of the bounding planes. After the feature is fit as a planar curve, it is projected back into 3D.

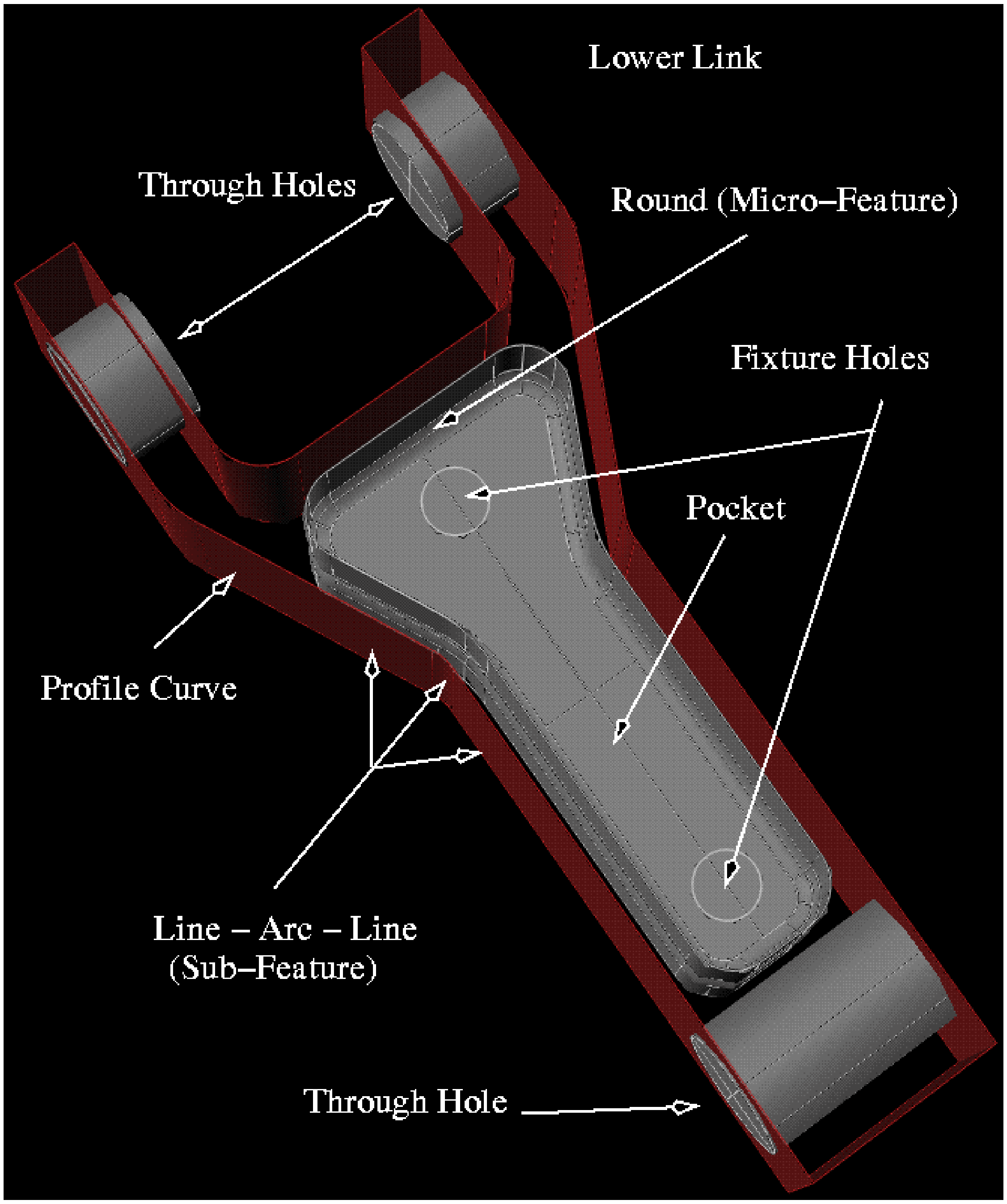


Figure 3-4: Feature-based Description

The reconstructed features should automatically be positioned in such a manner as to recreate, as far as possible, the properties intended by the part's designer. Failure to do so can result in a close approximation that still does not meet the tolerances required by the reverse engineered model.

The limited ways in which primitives can be combined to form mechanical features is in of itself a type of constraint on the possible geometry of an object. Enforcing global geometric properties across a single feature or between multiple features can augment these local constraints. For example, the designer has most likely placed the two pockets (seen in the Lower Link) symmetrically. Likewise the walls of the pocket have probably been placed in parallel to each other and the profile of the object. If each pocket is fit separately, it is unlikely that they will end up symmetric to or aligned with each other. Further, if each wall segment is fit separately, it is unlikely they would end up truly parallel. If this were a high precision part, such a change in the center of gravity could adversely affect wear and performance.

As another example, consider the upper holes on the Lower Link used to hold a connecting pin. Current sensing processes result in poor data for holes because they self occlude (especially small diameter holes) and have sharp discontinuities. If each hole is fit independently, no guarantee can be made that the assembly will still function. Even a small translation in the location of the center of high tolerance holes can prevent a mating piece from sliding smoothly between them. In these cases, the importance of constraining the centers of the holes and the common radii cannot be overstated. The result of this process enforces that the functionality of the holes (holding a mating pin) will be achieved, even if the sensed data contains error that would otherwise adversely affect this property.

When attempting to model the Lower Link the reverse engineer would notice several likely geometric properties (see Figure 3-5):

1. The 2.5D nature of the part suggests that the planar surfaces along the sweep direction are all aligned.
2. The upper holes appear to be concentric and have the same diameter.
3. The outer profile edges help shape the interior pocket contour they surround.

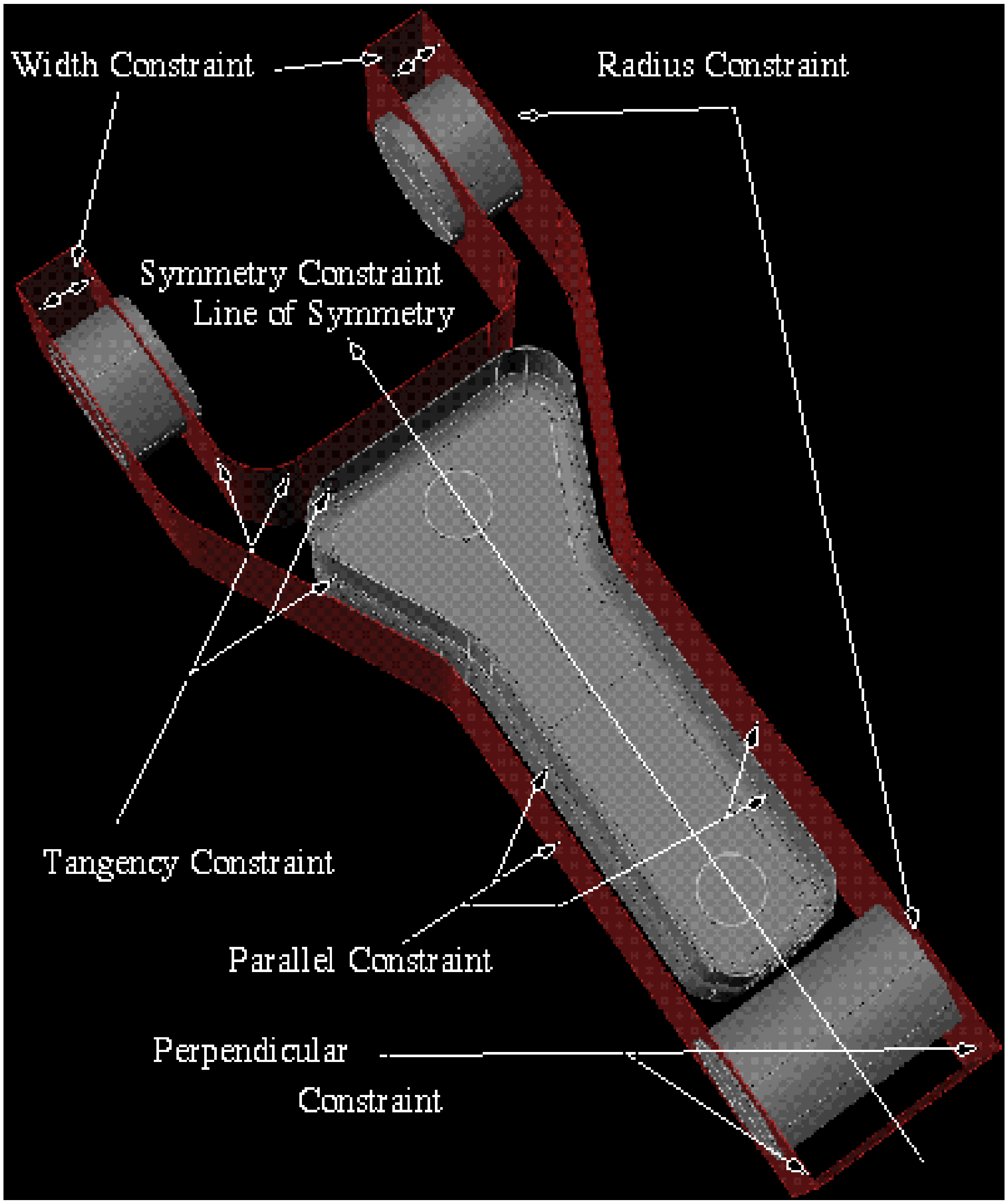


Figure 3-5: Constrained Lower Link

4. The pockets on either side of the object look as though they contain identical geometry offset along the part's sweep direction.
5. Many of the planar surfaces are parallel or perpendicular to each other.

A study of the Lower Link and similar parts makes it apparent that the geometry in this domain is not free form, but highly structured. This structure can and should be utilized during the model reconstruction phase. Constraints are a natural representation for capturing knowledge of the design process. Table 3-1 contains a list of constraints consistent with the design of 2.5D machined objects. These constraints represent characteristics implicitly and explicitly utilized during part design.

3.1.3 Lower Link Conclusion

A key insight about the geometry of 2.5D machined parts is that most, if not all, of elements are constrained externally by their neighboring features and internally by their own design. The lower link is sensed initially as an unordered point cloud. To create an appropriate model for manufacture and redesign, it is necessary to apply more information than just the sensor data.

Table 3-1: Identified Constraints

Incidence	Primitives (lines and arcs) meet at well-defined locations.
Smoothness	Primitives meet with equivalent tangents.
Perpendicularity Parallelism	Feature lines are often aligned parallel to each other or at right angle intersections.
Concentricity	The center of holes and arcs are often created to be concentric with one another.
Symmetry	Features are often created in whole or in part with symmetric relationships around specific axis lines.
Common Angles	Lines and planes often intersect at well-defined angles, such as 30, 60 or 90°.
Radius Consistency	Arcs often have identical radii when compared with other arcs in the contours of the object. Likewise, holes in an object are often drilled with the same bit or a limited set of bits.
Width Consistency	Pocket walls and other types of features often contain equivalent thickness throughout the entire part.
Identical but Offset	Certain geometries are found in one part of the object and repeated in other areas.

Knowledge of the design and manufacturing process can help produce low DOF models that can be further constrained by domain specific knowledge. The resulting models more accurately represent the likely geometry of the part than those produced by traditional fitting methods and contain topology that most likely matches that envisioned by the original designer.

3.2 Design and Manufacturing in the Domain of 2.5D Axis Machining

This research claims that certain geometries are much more likely to be found in the realm of 2.5D mechanical design because of the nature of such parts, and thus the geometric properties of these shapes should be exploited when reverse engineering them. To support this claim, the concepts of design practice, manufacturing knowledge, and design intent will be briefly examined.

1. Manufacturing Knowledge: 2.5D parts are usually machined using a three axis machining center. This machining process limits the possible geometry of a part. A block of aluminum (or other material) is placed in the milling machine, at which point a tool (i.e., end mill, drill, ream, tap, etc.) aligned with the Z-axis is used on the initial stock.
    - Typically a part has a 2D contour that is swept into the third dimension. Machining away all material on the outside of the contour physically reproduces this. For holes, the drill bit completely pierces the stock, producing holes aligned with the Z-axis. This machining process lends itself to extruded 2D geometry and planar surfaces.[18]
    - In the cases where off-axis geometry is needed, the part must be removed from the machine tool and then refixtured. This is usually accomplished by remounting the part along another planar surface that has been machined flat by the previous machining cycle. Such a surface is almost always orthogonal to the original

[18] Three axis machining can be adapted to produce certain free form pseudo 3D geometries, but this type of machining is seldom used in practice.

mounting surface. At this point, the same limitations of the three axis machining center apply.

2. Design Intent: Design intent can be broken into two forms, feature interaction and model appropriateness.
    - Feature interaction is the idea that the perceived interaction between features is by "design" and not by coincidence. A simple example would be that if two walls of a 2.5D part appear to be parallel, then it is likely they were intended to be parallel and should be fit as such.
    - Designers often choose the simplest geometry to accomplish their goals. Model appropriateness describes the ability of a model to represent this goal. This means that the fit geometry is consistent with the designer's intent. Hence even if a high DOF spline can better represent the sensed data for a certain feature, it would not closely match the designer's conception of the feature.

3.3 The Advantages Attained by Using Constraints

The use of consistent patterns in the design of 2.5D manufactured parts provides the foundation behind many design systems and the reverse engineering system presented in this dissertation. Regardless of the design system (which differ in implementation and representation) a designer uses, the results tend to be straightforward with well-defined characteristics. Thus, whether a designer uses a system based on non-uniform rational B-splines (NURBS), features, triangulated polygons, or even hand drawn lines, it does not alter the fact that the resulting design contain the same properties which can be leveraged during reverse engineering.

The power gained by using constraints is the ability to lower the DOFs of the problem, making the system more likely to converge to the desired result and less affected by noise. The following areas demonstrate how this advantage is realized: 1) enforcing design intent, 2) simplifying the fitting process, and 3) binding geometry together.

3.3.1 Enforce Design Intent

Design practice suggests that even complex models are often defined in terms of simple well-defined shapes. With no constraints applied, standard modeling systems allow the creation of many arbitrary shapes not seen in manufacturing. Even a simple constraint that forces segments of a profile to be incident to each other is necessary to enforce a real world property that is intuitive to the designer and reverse engineer. Thus the use of constraints can limit the reverse engineered model to the realm of feasible models produced by designers. This can be done in two manners. First, the choice of primitives can provide a powerful tool. Second, mathematical constraints can be applied over the primitives to further limit their DOFs.

3.3.2 Simplify the Fitting Process

The fitting process involves a weak search over a large vector of parameters. Any step that can reduce the number of parameters or the size of the search space speeds the search and reduces the effects of noise. Constraints can be used both for DOF reduction and to set feasibility boundaries on the search.

3.3.3 Binding Geometry

Traditionally, physically separate geometric elements are fit independently. This causes multiple fitting steps on smaller amounts of data and allows geometries that are linked by the design to "float" unrelated to their counterparts in the reconstructed model. When fitting the various geometries that make up an interesting part, it is common to find that they share certain geometric properties. This is often the case, for example, where several holes have the same radii or are concentric with each other, where the walls of a pocket have a constant offset from their counterparts on the surrounding profile, or where the diameter of a round is consistent throughout a model. Because of the errors associated with data sensing (notably noise and missing data), significant parametric errors can be introduced when fitting geometry. These problems can be addressed by the use of geometry spanning constraints.

Binding geometry across a single part can reduce the possible forms of the part and simply the fitting stage. Several examples were detailed using the Lower Link, including symmetrical pockets and aligned holes. Other examples include common wall thickness, symmetric halves of a part, and mating holes in an assembly. Binding geometry across the multiple parts of an assembly can be instrumental in enforcing tolerances and thus allowing assemblies of parts to fit with together.

3.4 Taxonomy of Constraints

When recreating a mathematical model of a manmade object from an exemplar part, it is beneficial to provide mathematical constraints that enforce known or suspected design artifacts. One way to achieve this is to use a limited set of features, with low DOFs, for modeling aspects of a part. This ensures that the optimization process does not fit just the noise inherent to sensed data, but also fits the inferred design intent of the object. Unfortunately, even simple features and subfeatures are not restrictive enough for the reverse engineering application. There is a need to further limit the possible geometry of the model. This can be accomplished by applying certain geometric constraints to the optimization process, limiting the feasible geometry of the part to those that are common in the domain of interest.

The previous sections described constraints in terms of local and global properties. This section presents a clarified taxonomy of constraints. Many 2.5D manufactured parts can be formed by combining a small set of domain specific primitives, termed features.[19] Features form the basic building blocks of design, representing such items as pockets, profiles, and holes. Features are composed of geometric primitives, notably lines and arcs. The design process suggests these primitives can only be combined in a few limited, well-defined ways. These composition characteristics are termed design pragmatics. A further level of control, termed functional constraints, governs the placement of features and subfeatures in relation to each other.

[19] From this point on, the term feature and domain specific primitive will be used interchangeably. Any use of the term primitive (as opposed to domain specific primitive) will refer to the line and arc geometry that make up the basic building blocks of pockets and profiles.

Figure 3-6 symbolically depicts the increases in accuracy gained from adding additional constraints and thus reducing the possible DOFs of the model.

3.4.1 Domain Specific Primitives

It has been shown by Thompson et al. [35] that features form a natural decomposition of 2.5D manufactured parts. The use of domain specific primitives such as those shown in Table 3-2 can greatly increase the accuracy and appropriateness of a recreated model. Features are based on common design practices and facilitate making simple accurate models. The fact that these features are used in design is a strong argument for their use in reverse engineering.

Features, such as pockets, in terms of 2.5D architecture, are formed from the combination of two bounding planes and a characterizing contour. Contours are formed from a collection of ordered primitives that fully define the 2D geometry of the feature. Without any limiting constraints these contours could contain arbitrary geometry. In practice a number of pragmatics are enforced. These pragmatics will be discussed in the following section.

Perhaps not as readily grasped is the idea that optimizing over features is simply another form of constrained optimization. Features were created based on common engineering structures, and thus map directly to design artifacts. For example, consider Figure 3-7, which depicts three separate optimizations to the same set of data. The piecewise linear interpolation produces zero error when compared to the data, but with the knowledge that the data points were scanned from a circular hole, a better representation can be chosen, constraining the fit to a circular shape.

A distinction is made between modeling error and data-fit error. Features help decrease modeling error and thus increase the accuracy to the original design, but when compared to the data, they increase the error over more general primitives.

Features also form natural decompositions of machined parts. They hypothesize the general shape and are useful in segmenting the scanned data into local regions. Further, features form a basis for applying global constraints across objects. This follows from their algebraic descriptions. Such global constraints will be discussed in the section on functional constraints.

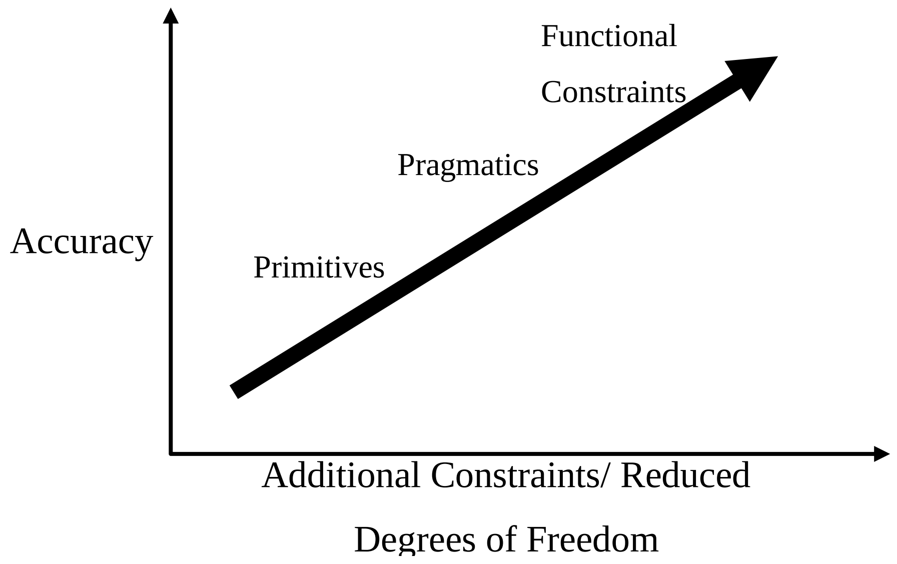
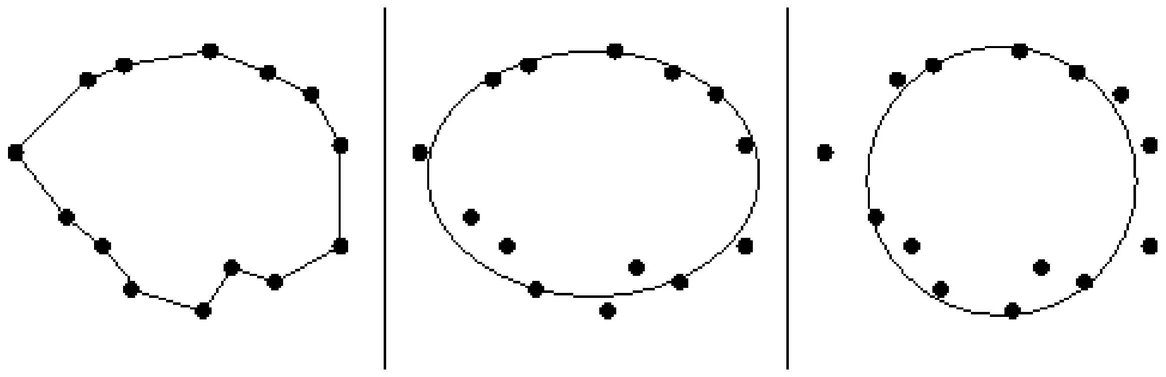


Figure 3-6: Increased Accuracy

Table 3-2: Feature Taxonomy

Feature	A high level geometric entity formed by combining subfeatures and microfeatures.
Holes	Blind, Through, Counter-sunk, Counter-bored, Tapped
Contours	Pocket, Profile, Island, Boss
Slots	
Primitives	The geometric primitives used to form features.
Planes	
Lines	
Arcs	
Micro Features	A manufactured geometric entity that is too small to be accurately identified by automated sensors.
Rounds	Smooth exterior surface between meeting planes
Fillets	Smooth interior surface between meeting planes



Fit:	Line Interpolation	Oval Fit	Circle Fit
Data Error:	Zero	Low	High
Design Error:	High	Medium	Low

Figure 3-7: Fit Options

Features are represented as tagged parameter vectors. At its most general, the feature vector for the contour pocket seen in Figure 3-8 would be composed of the following primitives:

Arc1, Line1, Arc2, Line2, Arc3, Line3, Arc4, Line4, Arc5, Line5, Arc6, Line6

The geometry of each primitive is represented as follows:

Arcs:	Center(x, y), Radius, starting theta, ending theta
Lines:	End 1 (x, y), End 2 (x, y)

An arc requires five values and a line segment requires four. The pocket profile is defined by 54 values (six lines and six arcs). Fortunately a great deal of simplification can be applied to these structures to reduce the number of parameters required to represent this geometry.

3.4.2 Domain Specific Pragmatics

Mechanical features have many implied restrictions on their possible geometry. Their representational power must be explicitly captured by the reverse engineering

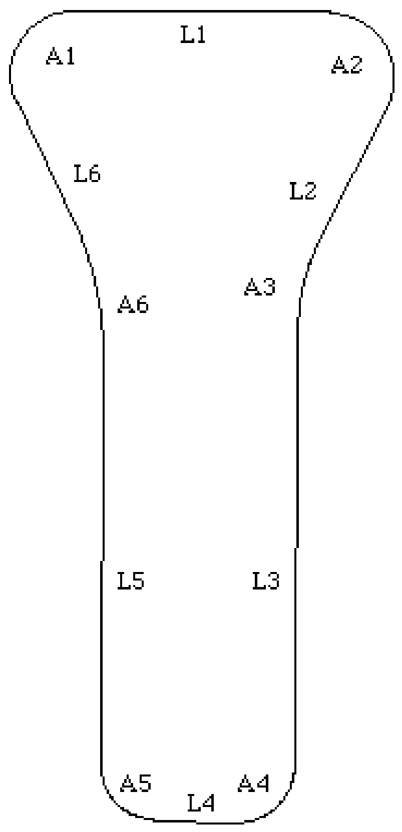


Figure 3-8: Pocket Profile

system in order to force the reconstructed model into a form that is likely to recreate the desired part. Primarily, only a limited set of primitives, as discussed above, is allowed to represent each feature. Second, the primitives can only be placed based on a rigid set of rules. These properties/restrictions are termed domain specific pragmatics and they include:

- Incidence - Connectivity of adjoining lines and arcs
- Smoothness and Continuity - Arc features usually form smoothing actions between line segments, providing C2 continuity

These pragmatics represent the intuitive idea behind how features are constructed and manufactured. They further allow the reduction of the DOFs necessary to represent features. Now line segments can be replaced with lines that require two DOFs to represent (the normal and distance from the closest point on the line to the origin). Arcs can be represented based on their surround geometry and as is described in the next chapter, require as few as one DOF. Pragmatics represent the implicit assumptions made by designers (and reverse engineers) when creating models.

3.4.3 Functional Constraints

Domain specific primitives model the recreated geometry in a manner similar to that of the design process. For 2.5D machined parts an appropriate choice of primitive is that of design feature. Domain specific pragmatics enforce even greater restrictions on the possible geometry of the features. For example, a domain specific pragmatic forces the use of continuous smooth contours. Functional constraints attempt to enforce high-level design intent.

When an engineer designs a part and chooses a specific feature, it is with the expectation of a certain functionality. An example would be the placement of symmetric weight reduction pockets to ensure a correct center of mass, or the placement of concentric holes to ensure the acceptance of a mating feature. With the error inherent in data scanning, small variations are likely to corrupt the recreated model. These variations can impact the usefulness of the replacement items.

Once again consider the Lower Link example in Figure 3-1. The line primitives that make up the profiles and pockets were not randomly placed but were purposely set at parallel and perpendicular angles to each other. Further, the top and bottom pockets as a whole were set directly opposite from each other and centered in the overall profile. The top and bottom mating holes were machined to accept similar connectors and thus are of the same radii.

All of these properties are designed into the object to promote its functionality, while maintaining an easily machined geometry. By enforcing these constraints, the DOFs of the hypothesized model are reduced and each separate feature is bound to the whole in a well-defined and enforceable manner.

This dissertation considers the effects of the following functional constrains: Perpendicularity, Parallelism, Concentricity, Symmetry, Known Angle between subfeatures, Arc Distance, Radius Consistency, Width Consistency, and Identical but Offset features.

3.5 Overview

Figure 3-9 demonstrates the progression of constraints used to guide the optimization of the bottom of one of the Lower Link pockets. As more information (in the form of constraints) is applied to the reconstruction problem, the less unintended-effects from sensor noise will bias the new model. In the following chapter the methods for representing these features and constraints will be discussed.

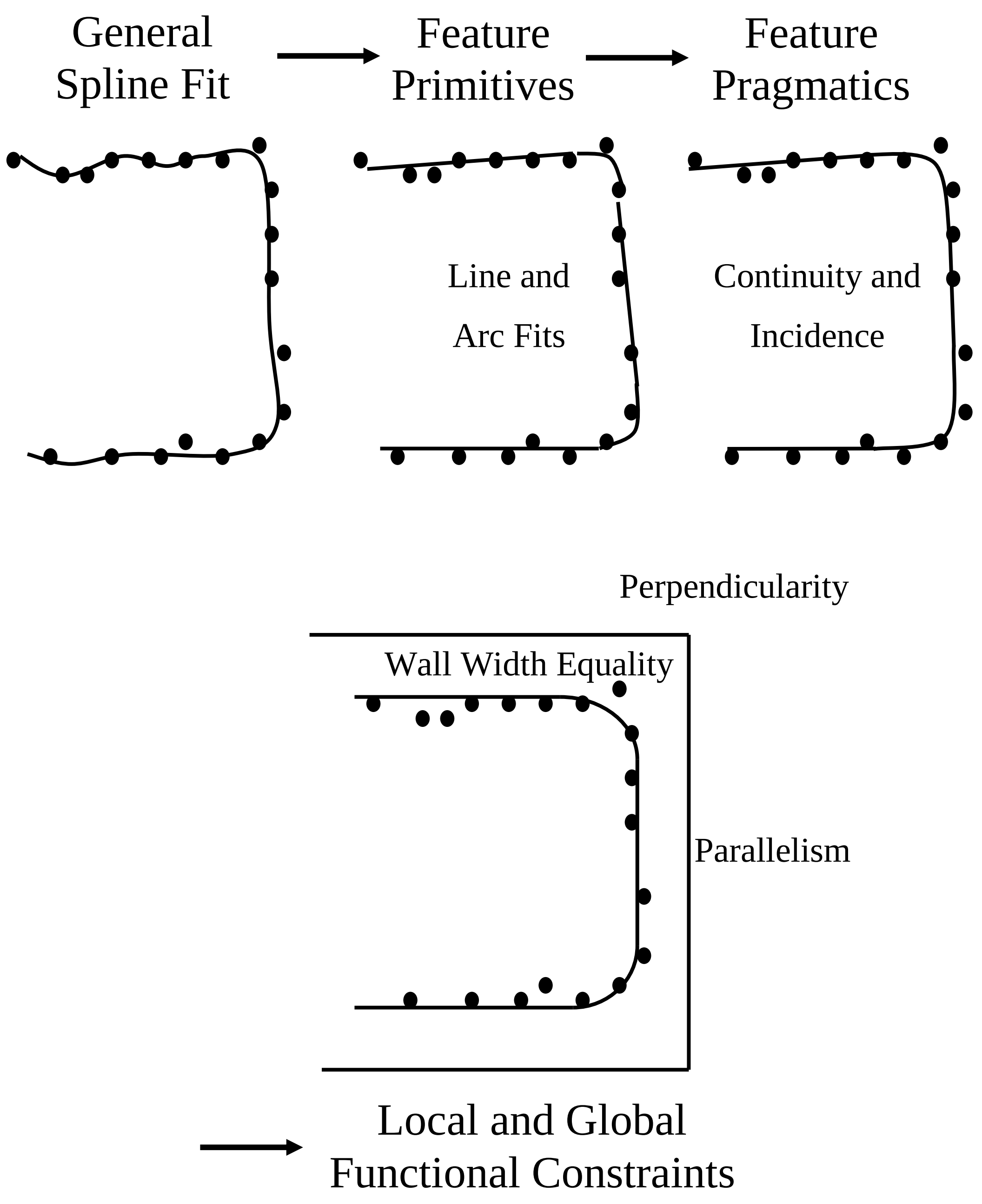


Figure 3-9: Constraint Progression

CHAPTER 4

CONSTRAINT FORMULATION FOR MODEL OPTIMIZATION

This dissertation claims that domain specific knowledge can be applied to the model creation process to generate higher quality models. In the previous chapters, the type of model and the properties inherent to the domain of 2.5D manufacture are detailed. This chapter describes the formulation of both the model and the constraints in such a manner as to be incorporated into an optimization algorithm. While this dissertation does not attempt to advance the core field of optimization research, it shows the ability to introduce domain specific knowledge via constraints into an optimization framework.

Each model is reconstructed based on a set of manufacturing features similar to those utilized in feature-based design. Holes are defined directly by their parametric geometry. Higher order features, such as pockets and profile-contours, are made up of low-level geometric primitives (arcs and lines). Each primitive has one or more parametric forms that can be used to construct the geometric form. These constructions[20] are made based on the current parametric representation, the actual parametric values, and ruler and compass composition algorithms.

All information necessary to construct the complete 2.5D geometry of the model is kept in a feature vector. A feature vector is defined as the list of all parametric values and topological information required to instantiate the model. The parametric values in

[20] The term constructive geometry is used here to describe the transformation from parametric space to geometric space using basic geometric rules (hence the term ruler and compass). Various parametric representations can be used to define a single geometric shape. For example, a line can be represented as a point and a normal, or as an angle and minimum distance from the origin.

the feature vector are the components that define the optimization space. The topological information contains which parameterizations are in use, as well as other book keeping information necessary to reconstructing the model from a minimum set of parameters. By adaptively changing the parametric values, variations of the shape of the object can be hypothesized and tested.

The initial feature vector for a part is constructed via an interactive system that fits low level geometry to the data cloud using standard least-squares fitting techniques. These initial fits of the low level primitives (circles, lines, and arcs) form the input to the constraint-based optimization process. First, the primitives are mapped onto features by enforcing the appropriate domain specific pragmatics. Then higher-level functional constraints are hypothesized from the low level geometry. Finally, penalty functions are asserted and the optimization begins.

The process of enforcing smoothness and incidence on the primitives is accomplished by intersecting adjacent lines and changing the radii of the arcs so as to merge with adjoining arcs or lines. This produces small deformations in the feature that are incongruent (as would be expected) with the sensed data. Under feature-based reverse engineering, an optimization would then take place on each separate feature, resulting in the final model. For the purposes of constrained optimization, the feature vector, representing the entire model, is optimized, accentuating or alleviating these deformations from the data to create a globally more appropriate and accurate model.

Once the initial model has been created, constraints are hypothesized and asserted producing a new feature vector representing the same geometry but with a greatly reduced number of parameters. Any constraints that cannot be directly represented through a parameter-set transformation are used to construct penalty functions to direct the optimization process. The output of this process is an instantiated parameter vector that most accurately (to the criteria of the constraints and sensed data) represents the form and function of the hypothesized original design.

4.1 Problem Formulation

The goal of the optimization process is to find the set of parametric values that produce the model that best fits the actual 3D position data while concurrently preserving the constraints asserted upon the possible geometry of the model. This problem can be formulated as seen in Table 4-1. **P** represents the vector of parameters defining the model geometry. **M** is a function that computes the geometric model based on the parameters **P**. **M** contains algorithms for transforming from the parametric space to the geometric space for each supported feature. **T** is a transformation function on p that commutes geometric constraints into a reduced set of parameters, $\mathbf{p_r}$ which is constructively equivalent[21] to the original, in that it can reproduce the same geometry as produced by **M**(**p**). D represents the error between the constructed model and the data points. E represents the violation of the constraints associated with the optimization. The goal of the optimization process is to minimize **F**, the weighted sum: $\mathbf{D} + \boldsymbol{\alpha} * \mathbf{E}$.

By defining these functions, the reverse engineering problem is transformed into an constrained optimization problem. The optimization is achieved by standard methods,[22] which, when applied to the parameter vector, pr produce the best model based on the error criteria.

This research has successfully met the following objectives:

1. The model creation function **M** was designed incorporating a representative set of 2.5D feature geometry. The function was implemented using constructive ruler and compass algorithms similar to those used in feature-based design packages.
2. The transformation function **T** was defined to reparameterize the model based on the constraints, reducing the number of DOFs while maintaining the same geometric representation. This function uses symbolic substitution methods and has an understanding of the geometry creation methods found in **M**.

[21] **T** also produces the book keeping information necessary for the transformation, such as the current parameterization used by each feature. Because this information is constant, it is not considered as part of the search space.

[22] For purposes of this work, both Simplex Search (amoeba) and Powell's method (conjugate directions) were investigated. Both methods were amenable to the formulation of the model and constraints used in this research and were able to converge to acceptable minimums.

Table 4-1: Problem Formulation

b:	Global book keeping information necessary for the reconstruction of the model.
C:	The constraints asserted on the model.
D:	The sensor data associated with the exemplar under reconstruction.
P:	Vector of parametric values used to instantiate the model geometry.
P_r:	A reduced DOF vector representing the same information as p based on the asserted constraints and B.
M(p,b):	A model creation function that constructs the physical geometry of the model from a parametric list of values and types.
$T(p,c)=P_r$:	A transformation function which reduces the number of parameters based on the constraint list c.
$D(M(P_r,b),d)$:	**D** is an error function that computes the distance from d to the model. (More precisely, this function must segment d based on the distance to the closet primitive and then compute the RMS error by summing the error over each primitive.)
$E(M(P_r,b),c)$:	Error derived from the violation of constraints, c.
F:	The object function to be optimized over. Equivalent to $\mathbf{D} + \boldsymbol{\alpha} * \mathbf{E}$. Alpha is a weighting term equalizing the data and penalty errors. The optimization algorithm.

3. The function **D**, which computes RMS error between the sensed data set and the current model geometry, was created. **D** also resegments the data during the optimization in order to limit the effects of outliers on the optimization.
4. A method was created for producing **E** directly from the list of constraints. This set of penalty functions was integrated into the objective function based on both geometric properties (e.g., distance between two lines) and parametric properties (e.g., radius equivalence).

The application of **T** to create the new parameterization vector is a preoptimization step that significantly reduces the complexity of the model under consideration, and thus the optimization process itself. **E** is computed during each step of the optimization based on constrained optimization techniques.

4.2 Constraint Representation

Features were introduced and formulated in Chapter 3. Consider a pocket feature comprised of a bounding plane, an extrusion depth, and a contour. The contour is an annotated ordered list of geometric primitives (viz., arc and line segments). Each part of the feature is defined by its parametric values which, when instantiated, determine the physical shape of the object. Under this formulation, many of the geometric constraints can be applied over the parametric representation.

The first application of constraints is to reduce the number of parameters necessary to define the model. $\mathbf{T}(\mathbf{p},\mathbf{c}) = \mathbf{p}_\mathbf{r}$. This is accomplished by replacing multiple parameters with a single parameter and a constructive method for recreating the lost parameters. The reasons for reducing the number of variables are:

- The constraints are implicitly enforced,
- the optimization convergence is faster,
- and the resistance to data noise is improved.

In the terminology of optimization, an interior point method has been created which searches for the best possible (lowest error) model while maintaining all constraints at each iteration of the optimization routine.

Because M constructs the physical geometry of the part based on ruler and compass techniques, and because different parameterizations can be used to represent the same geometry, it is not always possible to directly assert model constraints based on the parametric representation. In this case, the constraints are used to explicitly guide the optimization process by increasing the value of the objective function by an amount related to the degree with which each constraint has been violated. Function E(M(pr),c) is defined to calculate this result. Penalty functions force the optimization process to converge on a model with the desired topology. This is equivalent to an exterior point method, initially allowing infeasible configurations but then forcing the final configuration towards a feasible model.

4.3 Construction of Primitives

The low level elements that make up the 2D geometry are arc segments, line segments, and circles. For contours, domain specific pragmatics have shown that the geometry of the primitives is locally restricted by their neighboring geometry, and thus line segments can be represented as true lines, and arc segments can be represented as the curving action between lines. Primitive construction is the technique that constructively builds the physical geometry from the reduced DOF representations using ruler and compass techniques.

For example, consider two line segments that meet at a corner. If each line segment keeps track of the common endpoint, consistency errors can occur. If, instead, both line segments are transformed into line representation, then the common point can be calculated by intersecting the two lines. In this case, each time the parameterization of either line changes, the new end-point must be recalculated. For the constructive techniques involved in the features defined in the 2.5D domain, the complexity of the operation is constant.

Now, consider a circular arc bounded by two lines. As arcs are smoothing actions between two line segments, the arc must be incident and tangent to the neighboring lines. Therefore the arc can be constructed based on the line position simply by knowing its radius. Figure 4-1 represents the constructive geometry transforming the parametric representation into the geometric representation. The point **I** represents the intersection of the two bounding lines. By adding the normalized bisector vector times the distance **d** to the point **I**, the new arc center is determined. The intersection points between the lines and the arc are then determined by projecting the center onto both lines. This provides the complete geometric form of the line-arc-line structure.

In most constructive geometric operations, care must be taken to deal with singularities, such as if the lines become parallel in the line-arc-line example. In such a case, multiple constructive forms may be necessary and a transformation between them established. Alternatively, if the singularity can be hypothesized before the optimization

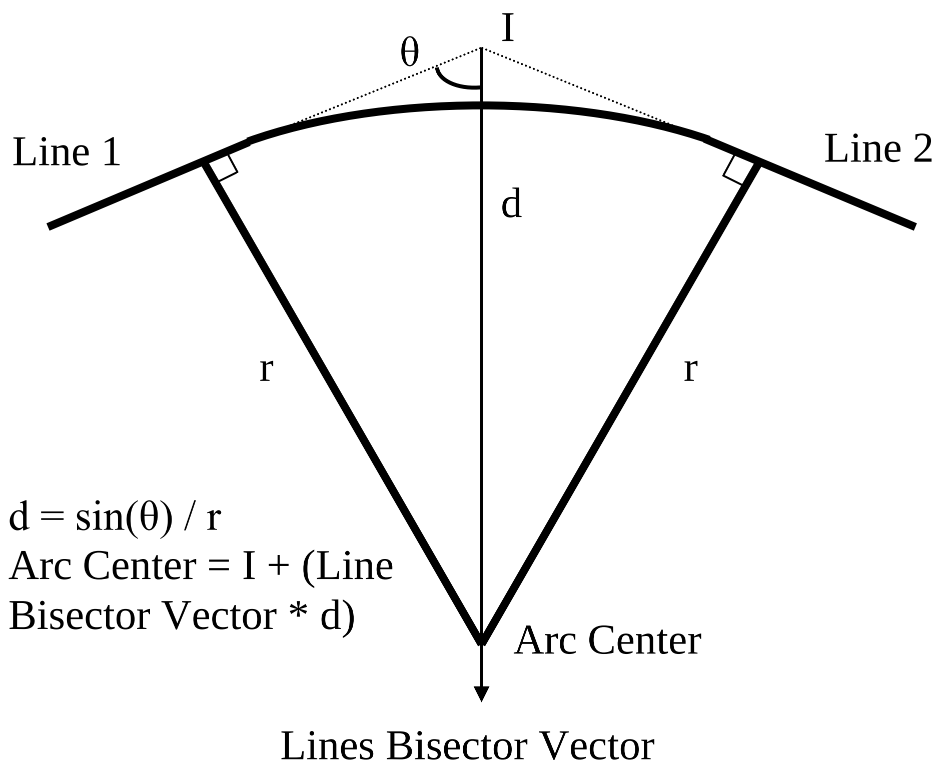


Figure 4-1: Arc Construction Based on Radius

(again as in identifying the parallel nature of the constraining lines) then only the parallel formulation need be used.

Another interesting feature found in the 2.5D domain is that of the boss, or triple arc subfeature (Figure 4-2). A boss often forms a strengthening structure around holes. It is composed of three arcs with C1 continuity. The radii of the outside arcs are equal, and the whole structure is symmetric based on the surrounding lines. Several parameterizations of this feature are required based on whether the bounding lines intersect, are parallel, or are collinear. This subfeature is an example of a domain specific structure that requires deep understanding of the domain, yet yields large dividends in the reconstruction process.

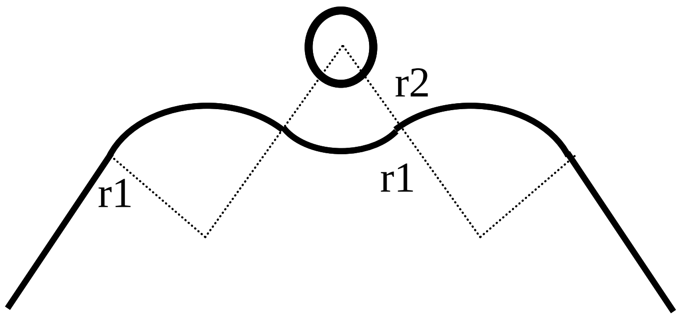


Figure 4-2: Boss (Triple-Arc) Subfeature

It should be noted that as the DOFs are reduced certain book keeping measures must be made, such as retaining the symbolic topological information (Line followed by an Arc, followed by a Line, etc.). In the case of the line-arc-line structure, there are four possible quadrants where the arc could fall based purely on the intersection of the lines. It is therefore necessary to store which quadrant represents the interior of the object. This allows the model construction method to recombine the reduced parameter model back into the full geometry of the part. This information is considered static throughout the entire optimization process and is not considered a DOF.

4.4 Defining the Reduced Degrees of Freedom Model

By using the asserted constraints on the geometry to produce the reduced parameter vector a powerful technique for constrained optimization is developed. The phrase DOF reduction is used for this technique. Two methods have been identified for reducing the DOFs of a model: symbolic substitution and feature re-parameterization.

4.4.1 Symbolic Substitution

As the name implies, symbolic substitution replaces equivalent symbolic parametric variables with a single common variable. Symbolic substitution is valid when primitives have parameters representing identical information, such as the radius of a hole, or the angle of a line.

Consider the two lines L1 and L2, defined by an angle, q, representing the line's normal direction, and the distance, d, representing the shortest distance from the line to the origin. If the lines are parallel, then this angle parameter can be redefined in terms of a single angle. Thus given:

Line1 $\Rightarrow$ (d1, q1) &
Line2 $\Rightarrow$ (d2, q2)

and the constraint:

Parallelism $\Rightarrow$ (q1 == q2)

a new description is generated:

Line1 $\Rightarrow$ (d1, q1) &
Line2 $\Rightarrow$ (d2, q1)

The result is a four-DOF problem being reduced to a three-DOF problem. For holes, defined by a center and a radius, symbolic substitution is possible for concentricity constraints.

Hole1 $\Rightarrow$ (x1, y1, r1) &
Hole2 $\Rightarrow$ (x2, y2, r2)

The constraint:

Concentricity $\Rightarrow$ (x1, y1) == (x2, y2)

The result:

Hole1 $\Rightarrow$ (x1, y1, r1) &
Hole2 $\Rightarrow$ (x1, y1, r2)

Thus a six-DOF problem is reduced to four. Likewise, for radius equivalence constraints, the single radius parameter can be substituted among all common radii holes, reducing the number of parameters necessary to represent each additional hole by one.

4.4.2 Feature Reparameterization

Unfortunately, symbolic substitution only works when two or more primitives have corresponding parameterizations (and thus construction methods). Consider a hole, represented by (x,y,r) and a bounding boss. A concentricity constraint cannot be directly applied because there is no direct representation for the center of the middle arc.

To address this shortcoming, it is noted that many geometrical forms can be parameterized in multiple way and still be instantiated to the same physical geometry. A line can be described as "Ax + By + C = 0" or as "y = mx + b" or as "(d1, q1)." Feature reparameterization is the modification of the representation of a primitive, giving it a new parameterization that can then be used for symbolic substitution.

The physical geometry for a linear edge on a model is represented by the line segment (x1,y1), (x2,y2). The slope of the line is not directly represented by this parameterization. By transforming the line segment into a line, the angle becomes available:

Line Segment(x1,y1), (x2,y2) $\Rightarrow$ Line(d1, q1).

At the same time the end points of the line are no longer directly available and must be computed by some new method (usually the intersection of the current line with the next line in the contour).

The choice of reductions is not arbitrary. In the arc example above, rather than calculate the start and end theta values for the arc segment by projecting the center of the arc onto the neighboring lines, the lines could be computed based on the angle of arc segment end-points. This formulation is poor because small fluctuations in these angles, which would be likely to occur because of segmentation errors, would cause large fluctuations in the line normals and would not necessarily maintain other line constraints such as parallelism.

For this work, the following ordering of primitives was used to decide which primitive required a new parameterization: lines, single arcs, holes, and triple arc features. Because of the general stability of the line parameterization and the widespread use of lines in this domain, most structures were based on the surrounding linear structure. When concentric holes and arcs occur, the hole parameterization is based on reconstructing the arc first and then equating the arc center to the hole center.

The result of symbolic substitution, feature reparameterization, and subfeature construction is the ability to apply the model definition function **M** on a set of fewer parameters while still producing a physical model equivalent to the initial model. The advantages of reducing the number of variables necessary to represent the geometry cannot be overstated and include superior noise resistance, more tractable problems, and faster and more accurate convergence.

4.5 The Optimization Function

Once the DOF reduction has taken place, the optimization process commences. All the constraints that cannot be symbolically reduced are imposed in terms of penalty functions during the optimization process. The objective function, **F**, is the weighted sum of **D** and **E**. The optimization can be defined as finding:

$$\min((D(M(p_r, b), d) + E(M(p_r, b), c)))$$

D calculates the sum of the square of the distances from each sensed data point to the appropriate primitive in the model. **E** calculates the violation of any constraints that have not already been symbolically formulated and represents the summation of the set of penalty functions. As described in the Chapter 2, penalty functions limit the feasible region of the optimization surface.

The following examples show the penalty functions used for concentricity and parallelism, where p_i represents the center point of each hole, and v_i represents the normal to each plane:

Concentricity: $f(p_1,p_2)$ = (geometric distance from p_1 to $p_2)^2 * k_i * w_i$

Parallelism: $f(v_1, v_2) = | 1 - (v_1 \cdot v_2) | * k_j * w_j$

where k_i and k_j are the controlling parameters and w_i and w_j are scaling parameters. The strength (magnitude of the value) of the controlling parameters is gradually increased during the optimization process to avoid initially falling into unrecoverable local minima, while gradually forcing the geometry into feasible regions as the optimization converges.

A difficulty found with penalty functions is normalizing them so that the error produced by one sort of constraint (say an angle based constraint) does not fade into insignificance or completely overpower a second constraint (say a distance criterion). The purpose of the scaling parameters is to adjust the value of each constraint into a level commensurate with the order of magnitude of the error function and with the other penalty functions. In this work, the penalty functions calculated scalar offsets (concentricity, width equivalence, radii equivalence, etc.). Thus the scaling between penalty functions was linear. The controlling factor of the penalty functions was increased incrementally over four separate runs of the optimization. The final value was three orders of magnitude greater than the initial value, forcing alignment errors of less than 0.1 microns.

4.6 DOF Reduction Example

Consider Figure 4-3, which depicts a simple part called the Steering Arm. The entire 2.5D geometry consists of two holes and an exterior profile containing five line segments and three arcs. To create the physical geometry for this object the following information is needed: for each line, the start and end points; for each arc, the center, the radius, and the start and end thetas; for the holes, the center and the radii. Specifically, the following geometric values are needed:

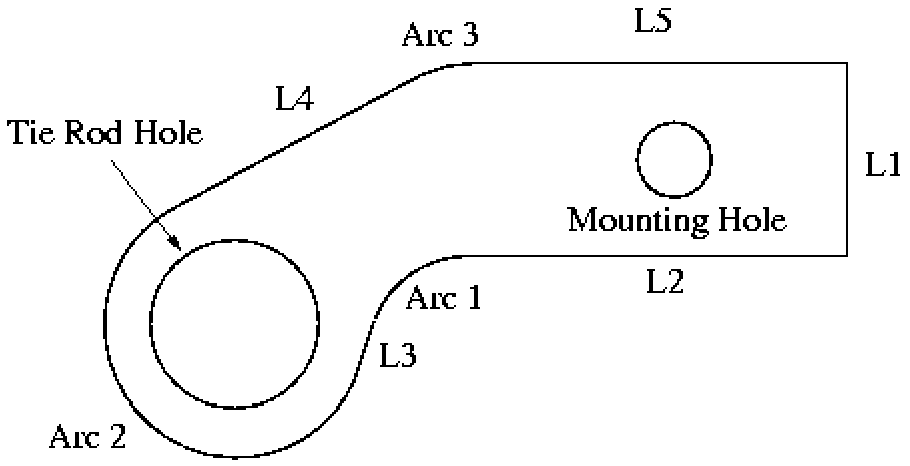


Figure 4-3: Steering Arm

Line1:	Start(x,y), End(x,y)
Arc1:	Center(x,y), Radius, ThetaA, ThetaB
Line2:	Start(x,y), End(x,y)
Arc2:	Center(x,y), Radius, ThetaA, ThetaB
Line3:	Start(x,y), End(x,y)
Line4:	Start(x,y), End(x,y)
Arc3:	Center(x,y), Radius, ThetaA, ThetaB
Line5	Start(x,y), End(x,y)
Tie Rod Hole:	Center(x,y), Radius
Mounting Hole:	Center(x,y), Radius

The above describes the geometry of the pocket in its most complete form and requires 41 DOFs. The following constraints are asserted on this geometry: The profile primitives are incident and continuous, the tie rod hole is concentric to the large profile arc, lines two and five are parallel, lines one and two are perpendicular, and the mounting hole is centered in the stock. The remainder of this section describes the transformation from this low level geometry to a feature vector utilizing a reduced parameter set that is still capable of constructing the geometry.

The first goal is to enforce incidence and continuity, reducing the number of parameters in the process. As detailed earlier, the arcs can be described as smoothing actions applied to the intersection of the lines. Therefore, the line segments can be redefined as lines (using an angle/distance representation) bounded by the arc segments:

Line1:	Theta1, Distance1
Line2:	Theta2, Distance2
Line3:	Theta3, Distance3
Line4:	Theta4, Distance4
Line5:	Theta5, Distance5

The constraint hypothesis step[23] asserts lines two and five as parallel to each other and perpendicular to line one. This allows the following symbolic substitution:

Line1:	Theta2 + 90°,	Distance1
Line2:	Theta2,	Distance2
Line5:	Theta2,	Distance3

Each arc can now be defined based on its radius and neighboring lines and the geometry constructed via the a fore mentioned ruler and compass method. This reduces the basic geometrical description of each arc from five DOFs to a constructive description requiring one DOF and insuring C1 continuity:

Arc1:	Radius1, Line2, Line3
Arc2:	Radius2, Line3, Line4
Arc3:	Radius3, Line4, Line5

The final hypothesized constraint is that the tie rod hole and arc two are concentric represented by the following substitution:

Arc2:	Radius2, Line3, Line4
Tie Rod Hole:	Center of Arc2, Radius4

Using incidence, smoothness, parallelism, perpendicularity, and concentricity constraints, the DOFs of the Steering Arm has been reduced from 41 to 15. By instantiating these parameters, the Steering Arm's geometry can be regenerated with these constraints implicitly retained. The final requirement is that the mounting hole must be located equidistant to Line2 and Line5.

[23] The next chapter details a strategy for automatically generating the appropriate constraints. For the purposes of this example, the engineer asserted the constraints.

4.7 Penalty Function Example

Due to small errors in the manufacture and/or the error associated with the scanning process, it is unlikely that the recreated mounting hole will end up exactly centered in the Steering Arm profile. A reasonable inspection of the part would suggest that this property was very likely to exist in the original design. To remedy this situation a constraint is asserted stating that the center of the hole is equidistant to the walls of the profile.

Consider the formulation of Line2, Line5, and the mounting hole from the Steering Arm.

Line2: theta2, distance2

Line5: theta2, distance2

Mounting Hole: center(x,y) and radius

There is no direct correspondence between the geometry or parametric representations of these features. Therefore it is necessary to construct a penalty function to account for this constraint. The error function of interest becomes the difference in distance from the center of the hole to the lines.

$$(\textbf{Distance}(\text{line2, center}) - \textbf{Distance}(\text{line5, center}))^2 * k_j * w_j.$$

During each iteration of the optimization process, this value is calculated and added to the error value associated with the current iteration. As w_j is gradually increased the center of the hole is forced closer and closer to the center of the two lines.

4.8 Discussion of Under and Over Constrained Geometry

In the case of constraint-based geometric construction systems (i.e., solvers), a large effort is made to deal with the problem of over and under constrained geometry. Under constraining implies that not enough information is present to fully describe the geometry of the object. Over constraining implies that more constraints are asserted on the geometry than are physically possible to maintain.[24] In the reverse engineering setting, these problems are not of the same concern for the simple reason that a real world instantiation of the geometry already exists.

First of all, a distinction must be made between the constraints imposed over the geometry on the model, and the "constraint" which is the sensed data itself. The tools for creating the initial feature vector only allow geometry that has a physical representation. Therefore the initial geometry can be considered well constrained.

Because the geometry of the part is well defined before the constraint optimization process and at least some data points are available for all primitives of the part, it is impossible to under constrain the reverse engineering system. The geometry is in fact always constrained by the physical data when no higher-level constraints are asserted.

The only concern is when the system (or user) provides a set of assertions on the geometry which create an over constrained set. In the case of symbolic substitution, the last substitution to be made will overrule previous substitutions. This results in utilizing one constraint and removing another. In most cases where this happens, the physical geometry of the recreated model is obviously inconsistent with the data, thus quickly allowing the engineer to identify the illegal constraint and remove it.

In the case of constraints represented by penalty functions, the numerical methods will attempt to minimize the error, resulting in an averaging of the inconsistencies. Again, this should be immediately apparent to the engineer via physical inspection of the recreated model and analysis of the final RMS error associated with each feature.

In summary, over and under constrained geometry is a difficult problem in the area of geometric solvers, but because a physical instance of the part exists for the reverse engineering process, it is not possible to under constrain the problem, and an over constraining of the problem results in immediate and apparent discrepancies in the topology of the model and the error associated with the features representing the model.

[24] An over constrained set of assertions, which can be reduced to a well-constrained set because one or more constraints are interdependent, is not considered over constrained.

4.9 Constraint Formulation Summary

Constraints are formulated directly on the parametric representation of the geometry either by appropriate substitution or by penalty function creation. Table 4-2 lists the identified constraints and their reformulation for use with the optimization process.

Table 4-2: Local and Global Constraints

Incidence	
Line to Line	Intersection forming corner.
Line to Arc	Projection of arc center onto line to form arc segment end point.
Arc to Arc	Any case of arc to arc incidence is handled by special constructive subfeatures, such as a boss.
Smoothness	
Line to Arc	Arcs are formulated as smoothing actions on lines based on the radius of the arc.
Arc to Arc	As above (arc to arc).
Perpendicularity	
Line / Line	The angular value of the first line is symbolically substituted for the second line (the value is then adjusted 90°)
Parallelism	
Line / Line	The angular value of the first line is substituted for the second line.
Concentricity	
Arc / Arc	A Penalty function is constructed based on the calculated distance between the two centers.
Arc / Hole	The arc center is substituted for that of the hole.
Symmetry	Symmetry is achieved by a combination of penalty functions and other constraints.
Radius Consistency	
Arc / Arc	The radius of one arc is substituted for the next.
Arc / Hole	The radius of the arc is substituted for the hole.
Width Consistency	
Lines / Lines	A Penalty function is calculated based on the square of the distance between the first set of lines minus the distance between the second set of lines.

CHAPTER 5

AUTOMATIC CONSTRAINT ACQUISITION

The advantages of and methods for applying constraints during the fitting process have already been discussed. The assumption has been that the constraints are known a priori to the global optimization phase, and that these constraints are accurate and meaningful. The important question remains: How were these constraints identified? Manufacturing and design knowledge implies not only how certain constraints should be enforced mathematically, but also which constraints are likely in the domain of 2.5D manufactured parts. This knowledge provides two insights. First, a well-defined list of possible constraints over each feature (or its associated primitives) can be identified. Second, a test for likely constraints can be implemented based on approximate equal parametric values determined during the initial fittings. Below is a list of the steps employed by Owen et al. [24].

1. Scan part to produce data cloud.
2. Manually segment bounding plane data and then fit individual plane.
3. Manually segment single feature data.
4. Fit data to lines and arcs based on local feature constraints.
5. Repeat steps 2-4 until all features have been identified.
6. Build model on a feature-by-feature basis.

These steps are augmented as shown below. Step five is the subject of this chapter, and steps six and seven are the subjects of the previous chapters.

1. Automatically segment and fit bounding planes.
2. Constrain the bounding planes based on alignment constraints.
3. Semi-automatically segment and fit features.
4. Identify likely constraints.
5. Build optimization function over all features enforcing constraints.

6. Optimize over all features simultaneously while maintaining constraints.

5.1 Manufacturing and Design Knowledge

Manufacturing and design knowledge can be used to formulate likely constraints that can apply to the specific features in a domain. As detailed in Chapter 3, certain geometric topologies and relationships are implicitly and explicitly used in the design and manufacturing of the 2.5D parts considered here. Some of these qualities are enforced during the reverse engineering process by the use of features that mimic the design patterns of the object. Other properties must be explicitly denoted by the reverse engineering system.

The first application of constraint acquisition is based on the implied qualities of three axis machining. In the case of these parts, the planar faces bounding an extruded 2D curve are almost always parallel to one another. This quality is implicit both in the designers sketch and in the CAD representation. During the reverse engineering process, a decision must be made on how to handle this and similar situations.

Consider a pocket bounded by bottom and top planes. Previous methods have either fit both planes individually or only used one plane as the basis for the feature. This leads to problems in calculating the height of the pocket as well as problems projecting the data into two dimensions for fitting. Manufacturing and design knowledge promotes the idea that the top and bottom planes should be fitted simultaneously with equivalent normals. This results in parallel planes. Further, in the case where one bounding plane is more accurately sensed (due to larger surface area or less occlusion) the optimization function can be weighted to constrain the less accurate section of data to the more accurate section. The result is a well formed bounding region for the feature of interest.

Additionally, problems arise when multiple features share the same bounding planes. If it were the case that only data points near the feature were used for fitting the bounding plane, or that the bounding plane was fit several times with different segmentations of data, then multiple planes would most likely be introduced into the new model where only one true plane existed before. High-level knowledge requires all shared bounding planes be fit simultaneously with all hypothesized parametric equivalencies enforced.

5.2 Parametric Equivalence

Parametric approximate equivalence is a method for identifying constraints on a specific model. Given two primitives (fit directly based on the sensed data), of types **α** and **β**, a cross reference is made to a correspondence table denoting the possible constraints which can hold between the given types of geometry (See Table 5-1).

In the case of two holes, the centers can be concentric, or the radii can be equivalent. In the case of two lines, the normals can be parallel or perpendicular, or in the case of parallel lines, they can have a common distance from another line. For every set of two primitives, the possible constraints are checked and a parametric equivalence test is made. This test takes the form of a distance check for concentricity, a length test for radii, or a theta check for parallel and perpendicular constraints:

Concentric Test:	$\|\|center1 - center2\|\| < \boldsymbol{\delta 1}$
Parallel test:	$\|theta1 - theta2\| < \boldsymbol{\delta 2}$
Perpendicular test:	$\|theta1 - theta2\| - 90 < \boldsymbol{\delta 3}$.

If a primitive is found to have a parametric value closely matching another primitive, then a constraint is hypothesized suggesting that any error between the two was caused by noise in the sensing process and a constraint needs to be enforced during the optimization.

Parametric equivalence is a powerful tool for speculating on likely constraints. These tests are valid because they are based on knowledge of logical design practices. For example, a typical part with holes will have a minimum number of different radii. In the case of threaded holes, the screws for one hole are quite likely to fit another hole.

Table 5-1: Example Constraints

	Hole	Line	Arc	Plane
Hole	Concentric Diameter Equiv.	Distance to Center of Hole	Concentric	-
Line	-	Parallel Perpendicular Distance to Line	-	-
Arc	-	-	Concentric Radius Equivalence	-
Plane	-	-	-	Parallel Perpendicular

Further, the radii of these holes are likely to be one of a set number of well-known drill bit sizes.

Consider Figure 5-1 depicting both a profile and a pocket feature. The values in Table 5-2 and Table 5-3 were computed fitting each separate primitive to its individual segmented data. The lines are parameterized in terms of their normal and distance from the origin. The arcs are parameterized based on their center and radius. By cross-referencing every primitive, several relations can be observed.

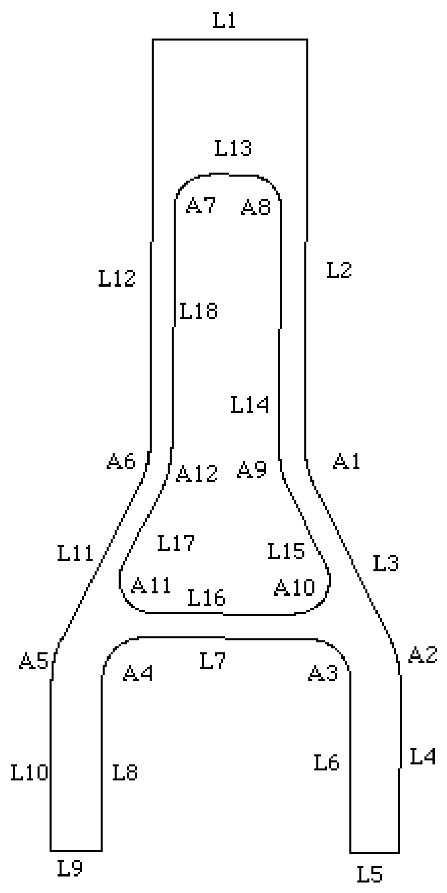


Figure 5-1: Lower Link Profile and Pocket Features

Table 5-2: Lower Link Lines Parametric Values

Lines	Normal in Degrees	Distance from Origin
L1	3.135	2.636
L2	1.566	-0.548
L3	1.107	-0.443
L4	1.565	-1.172
L5	3.140	-2.612
L6	1.563	-0.843
L7	3.136	-1.239
L8	1.566	0.752
L9	3.142	-2.620
L10	1.564	1.078
L11	2.029	0.351
L12	1.565	0.451
L13	3.122	1.759
L14	1.563	-0.402
L15	1.106	-0.300
L16	3.133	-1.077
L17	2.026	0.191
L18	1.566	0.288

Table 5-3: Lower Link Arcs Parametric Values

Arcs	Center (x)	Center (y)	Radius
A1	-0.020	-1.131	0.583
A2	1.507	-0.575	0.604
A3	1.504	-0.585	0.269
A4	1.521	0.469	0.280
A5	1.523	0.458	.610
A6	-0.007	1.025	0.571
A7	-1.536	-0.170	0.220
A8	-1.550	0.088	0.208
A9	0.034	0.807	0.519
A10	0.862	0.397	0.212
A11	0.858	-0.527	0.215
A12	-0.001	-1.014	0.607

First, lines L1, L5, L7, L9, L13, and L16 have normal orientations within 0.02 degrees. Likewise, lines L2, L4, L6, L8, L10, L12, L14, and L18 have similar orientations, as do lines L3 and L15, and lines L11 and L17. As these values are very close, they are automatically flagged as likely parallel constraints. Further, lines L5 and L9 have similar normals and distance parameters, implying that they are in fact two line segments on the same geometric line.

In addition to the parallel constraints, it is numerically apparent that many of these lines have normals perpendicular to each other within a very small error delta. Thus many of the lines, such as L1 and L2, can be flagged denoting a perpendicular constraint. The same process is applied comparing the radii of each arc. The result is the flagging of A7, A8, A10, and A11 as likely radius equal constraints.

5.3 Hypothesis Based on Geometric Considerations

As stated in the previous chapter, not all information about the geometrical relations of a model can be construed directly via the parameterization. For the case of identifying equivalent-width constraints a geometric test must be made.

Once again consider the Lower Link. An additional test is made calculating the distances between all sets of parallel lines. This information is saved in an upper diagonal matrix. After all the width computations are made, corresponding values are grouped. This allows an automatic assertion of width-equal constraints, such as usually found with pocket webbing or symmetric features.

Table 5-4 displays the matrix for a subset of the lines from the Lower Link profile and pockets. Two things are directly apparent: 1) the distance between L2 and L12 is almost exactly one inch which implies a distance constraint, 2) the distances between lines L2 and L14, L3 and L15, L7 and L16, L11 and L17, and L12 and L18, are all within 0.02 inches. This information can be taken for evidence that the pocket feature is centralized (on three sides) to the profile feature.

Similar to the line examples, several numeric equivalencies can be made on arc subfeatures. Arcs A2 and A3 and arcs A4 and A5 have centers within 0.015 inches. Similarly the centers of arcs A12 and A6, and arcs A1 and A9 are separated by several

Table 5-4: Line Distances

	L2	L3	L7	L11	L12	L14	L15	L16	L17	L18
L2	-	X	X	X	0.999	0.146	X	X	X	0.836
L3	-	-	X	X	X	X	0.143	X	X	X
L7	-	-	-	X	X	X	X	0.162	X	X
L11	-	-	-	-	X	X	X	X	0.160	X
L12	-	-	-	-	-	X	X	X	X	0.163
L14	-	-	-	-	-	-	X	X	X	0.690
L15	-	-	-	-	-	-	-	X	X	X
L16	-	-	-	-	-	-	-	-	X	X
L17	-	-	-	-	-	-	-	-	-	X

tenths of an inch.[25] Based on the scale of the object and accuracy of the sensor, the user would set the appropriate distance criteria to assert concentricity. Because access to the original Lower Link model is available, it is possible to determine that the first set of arcs is in fact concentric while the second group is not. In the case where the delta value is too large, a false positive result will be reported. This result can either be overruled by the user, or ignored, producing an erroneous constraint, yet one unlikely to adversely affect the final model.

5.4 A Word of Caution

It should be noted that parametric equivalence is not without its liabilities. There are several reasons that can cause the assertion of a false constraint. (1) The initial segmentation and fitting of certain small arc primitives can lead to vastly differing parametric values, even while maintaining a close geometric approximation to the correct shape. (2) The reverse engineer may fit a different feature to the data from that initially created during the design, causing a mismatch of parametric representation. (3) The apparent equivalence could simply be a coincidence in the data or the result of machining or sensor error.

Only in case (1) is it likely that the asserted constraint will cause dramatic, or even noteworthy, errors in the new model. In most cases the differing parametric values will cause a constraint not to be hypothesized. In the case of a false positive constraint, the produced model will usually be either so far off as to immediately draw the attention of the engineer, or so close that the change is unlikely to have a pronounced effect.

Of further note, the optimization process produces an error value associated with each feature. Any extreme peak in a single features error is strong evidence of improper constraints. (It should also be noted that large errors could occur when true constraints are dragging the model away from very poor scanned data and toward the desired geometry.)

[25] It is sometimes the case that coincidence does exist in model creation. Seldom will a designer purposefully offset a structure by hundredths or thousandths of an inch, but certain CAD packages will allow diverse feature parameterizations that can occasionally lead to these situations.

These results highlight two important ideas:

1. Parametric equivalence is a powerful tool for hypothesizing likely geometric properties of an object, removing the tedious job of asserting multiple straightforward constraints from the user.
2. If the initial fit is off because of noise, segmentation error, or other error sources, parametric equivalence can fail. Therefore, although this method is generally accurate, an interactive user should verify all constraints and may be required to assert missed constraints.

CHAPTER 6

THE REVERSE ENGINEERING PROCESS QUANTIFIED

There is a hierarchy of possible goals that may be desired for any given reverse engineering process. At the lowest level, a simple representation of the spatial geometry of the exemplar part must be created. Such a representation is inadequate for all but the most basic analysis and renderings. A feature-based reverse engineering paradigm provides not only a more useful and powerful representation, akin to that created by the modern design process, but also provides a method for noise reduction and error correction. Constraint optimization over features is the next logical step in the continued progression of this research.

Constraint-based optimization in the area of reverse engineering provides a method for creating high precision models that reflect the design intent behind the geometry of the exemplar part. This method enforces that the geometry created does not blindly fit the sensed data (which is known to have systematic errors, missing data, and noise) but also adheres to high-level design constraints. This reduces the sensitivity of the process to the errors introduced through machining, wear, and scanning.

This chapter discusses the entire engineering process as utilized in this work. To validate the approach, several exemplar parts have been chosen from those making up the University of Utah Mini-Baja and Formula SAE Race Cars (See Figure 6-1). Each part has multiple features, including holes, pockets, and profiles. Unconstrained, these models would contain from dozens to hundreds of DOFs. Such large optimization spaces are in the general case unsolvable due to myriads of local minima. By combining strong local starting points, DOF reduction, and constraint-based optimization tools, higher accuracy models for each exemplar part have been created. These models better capture the

Figure 6-1: Formula SAE Race Car

physical geometry and the functional properties of the object by using dependencies asserted across the entire object to bind the global geometry of the object. Such a model is of high value to the redesign process that often follows reverse engineering.

The remainder of this chapter details the stages of the reverse engineering process, how constraints were applied to the models and the results obtained. Certain stages of the process that are necessary to the overall system, but not the focus thereof, are only given cursory descriptions. The reader is encouraged to keep in mind the goals of this research, as outlined below:

1. To create high precision models that reflect not only the geometry, but as importantly, the design intent behind the exemplar parts.
2. To demonstrate that reverse engineering can be achieved and couched in the terms of constraint optimization.
3. To show that design and manufacturing knowledge can be codified as constraints to guide a global optimization process, thus producing higher accuracy models.

4. To automatically hypothesize and assert constraints based on knowledge of the design process.
5. To rapidly and semi-automatically segment and fit unordered data clouds, producing CAD models.

This work has demonstrated the ability to use optimization to produce high precision models while maintaining geometric properties defined by constraints representing real world "truths." Further, a method has been developed for automatic local and global constraint/dependency assertion. A rapid feature-based segmentation and initialization system has been developed to process unorganized data clouds and instantiate the optimization process.

6.1 Desired Results

The goal of each specific reverse engineering process is dependent on the projected use of the generated model. The techniques proposed by this dissertation are necessary because of the difficulties in sensing 2.5D mechanical parts to a sufficient level of accuracy for reproduction and the problems in producing models that are faithful to the original design. These same techniques are feasible because of the inherent structure in this domain.

To show the effectiveness of constraint-based reverse engineering, the recreated models were compared with the original models (available from the designer) used to machine the exemplar parts. The exemplars were manually measured to determine any manufacturing errors, and this information was used to assert the ground truth.

To quantify this research two separate groups of metrics are provided. These include:

Model Analysis:

1. Parametric Error in Terms of Design Intent
2. Volumetric Error

Method Analysis:

3. Automatic Constraint Assertion
4. DOF Reduction

Parametric error describes the differences between salient feature elements, such as how equal two radii are, how close the center of a hole is to a concentric hole, or how parallel one line is to another line. The parametric information is arguably of equivalent value as the geometric shape of the final model. Design intent is a qualitative comparison between the recreated model and the original model based on the quantitative parametric differences. The focus here is placed on how well the reverse engineered features represent design truths associated with the original model.

Volumetric error concerns the spatial difference between the reverse engineered model and the original model. The reported values show the square root of the mean of the squares (RMS error) of these values. Section 6.1.1 describes the process for generating this metric.

Automatic constraint assertions refer to how well the constraint hypothesizer has determined the constraints that accurately reflect the design intent of the object. This includes the number of false positives (wrong constraints) and false negatives (missed constraints). These constraints, along with those asserted by the reverse engineer, provide the design knowledge that drives the optimization process.

DOF reduction represents the number of parameters that have been removed via the automatic DOF reduction algorithm. The lower the number of parameters used to represent the model, the more resistant the optimization becomes to noise and local minima. This category also describes the number of penalty functions enforced on the model.

For each exemplar part, a description is given describing the part and the intent behind its design. The total DOFs associated with the part are shown along with the number left after DOF reduction has taken place. The effectiveness of the automatic constraint assertion process is detailed. A parametric analysis is given showing the feature-based errors encountered in relation to the scanned data and the original model. Finally a volumetric error measure is given, stating the total volume the final model differs from the original model.

6.1.1 Volumetric Error Calculation

The process of quantifying the difference between two nearly identical models is complex. As previously mentioned, discrepancies in parametric values, such as the radii of corresponding holes, are useful (even crucial) for item by item checking, but do not characterize the overall similarity or difference between the geometry of the two models. The volumetric error attempts to quantify the difference that would be found between the two models if they were overlapped and one subtracted from the other. Thus the volumetric error (**VE**) can be defined as:

$$\mathbf{VE} = \text{Volume}\ (\mathbf{M_o} - \mathbf{T}\ (\mathbf{M_r}))$$

Where **T** is the transform that minimizes the integral of the distance between corresponding surfaces of each model (i.e., the volume of difference), $\mathbf{M_o}$ is the original model, and $\mathbf{M_r}$ is the reconstructed model, and "**-**" is the Boolean operation differencing the two models.

Unfortunately, an analytic evaluation of **T** is not available, nor is it easy to compute the volume of the difference between two arbitrary solids. To address these problems an iterative optimization process using the 2.5D planar nature of the parts is used to approximate **T**. Once registered, the difference volume between the two models is calculated by a discrete sampling method.

The direct 3D registration process is difficult and expensive to compute. Fortunately, due to the 2.5D nature of the models, several advantages present themselves for the registration process. First, the bounding (or anchor) planes of the models can be directly aligned[26] and the difference in heights averaged. This changes the registration problem from a 3D model-to-model problem into a 2D contour-to-contour optimization involving only a single translation and rotation. Further, only the outer profile is used based on the fact that it is the most easily, and thus most accurately sensed feature of the model.

[26] In the case where a model has bounding planes with differing normals (nonoptimized models) the average normal is used.

Differencing two continuous curves is not easily accomplished. An approximation is made by taking a sampling of equidistance points (an arc length parameterization) along the length of one profile and computing an objective function based on the sum of the squares of the distances from each point to the nearest subfeature on the corresponding contour. This involves a segmentation process (corresponding each point with the nearest subfeature) and 2D distance calculations. The resulting optimization computes the estimate for **T** allowing for the volumetric differencing.

Once the models are registered, the volumetric difference between the two models is calculated. This is achieved by approximating the continuous nature of the surfaces of the model with a discrete sampling of points. Again using the nature of the 2.5D parts, the 3D geometry can be sampled by taking equally spaced points along each 2D profile (as generated during the registration process) and projecting them along the Z-axis at the same interval. This covers the sides, pockets, and holes with a set of surface-wise equidistant points, each representing approximately the same area of the surface of the model. The planar surfaces are sampled through a Cartesian grid of points. By creating a fine distribution of points, the continuous surface of the model is approximated.

Given an even distribution of points, the volume between the models is closely approximated by projecting each point from the sampled model on to the closest surface of the reconstructed model and multiplying this value by the area represented by the point. Because the related features for which each point was created are known, the error metric can be given on a feature-by-feature, subfeature-by-subfeature, or model-by-model basis.

6.2 The Parts

This work has been validated on a set of representative 2.5D exemplar parts[27] (see Figure 6-2 and Figure 6-3). The simplest part under consideration is the Steering Arm. It consists of a profile contour, a tie rod hole, and a single mounting hole.

[27] The design and production of the parts was done by mechanical engineering students and faculty for the annual Society of Automotive Engineers (SAE) design competition at the University of Utah.

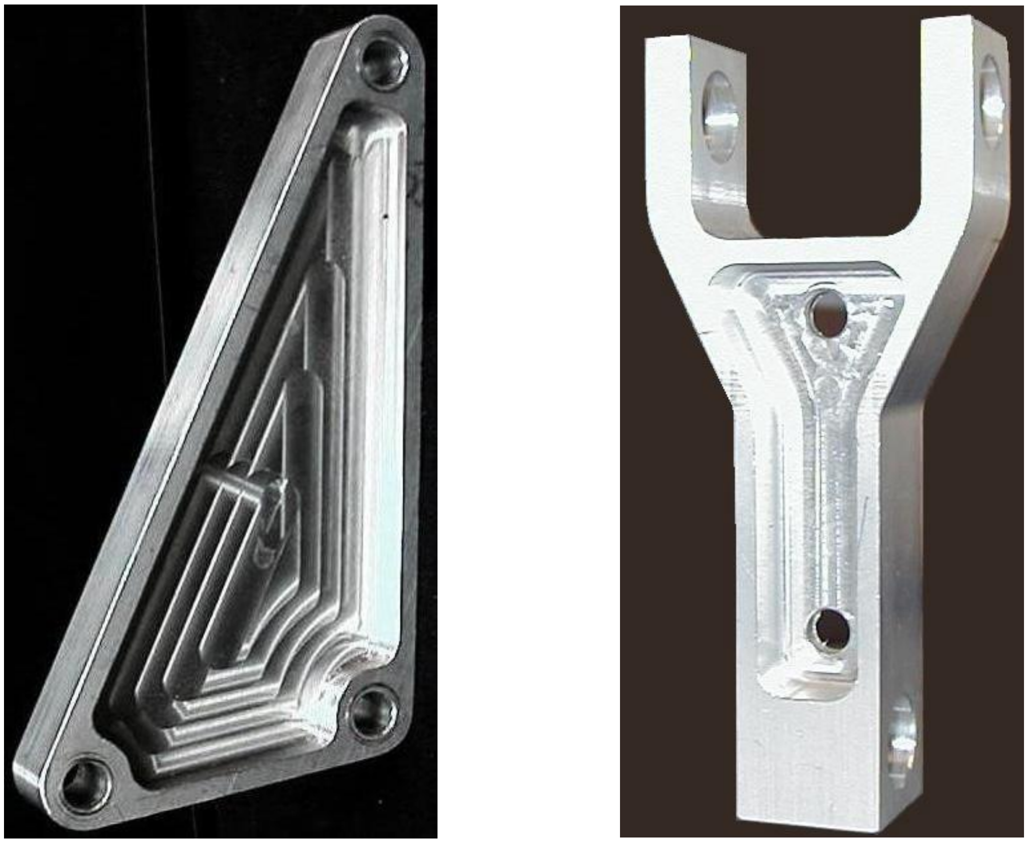
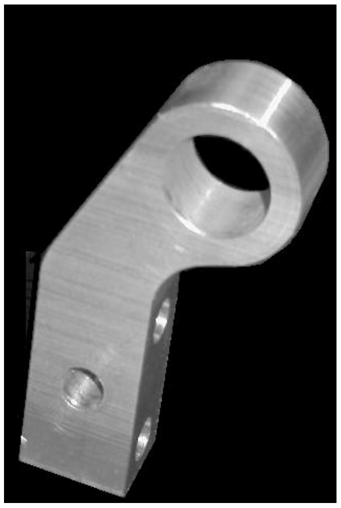
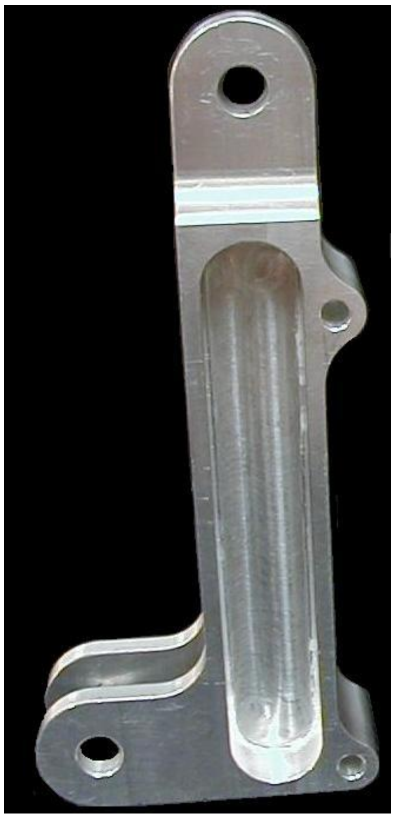


Figure 6-2: Shock Plate (left) and Lower Link (right)

(a) Upright



(b) Steering Arm



(c) Shock Tower

Figure 6-3: Additional Exemplars

This part demonstrates the basic and fundamental application of constrained optimization to improve the reverse engineering process.

The Shock Plate consists of two complex interior pockets, three through holes, and a simple exterior contour. It mounts to a shock absorber and wheel suspension and forms a rocker between the two. The pockets are symmetric and were designed with equal thickness between each wall and the exterior profile. The holes were designed to accept equal sized fasteners.

The lower link is also used as part of the wheel suspension. The lower link consists of an extruded profile, two pockets, and three through holes. The pockets are for weight reduction and are symmetric, with equal web thickness on three sides. The holes form assembly connections. The upper holes are aligned to accept a connecting rod. The lower hole has the same radius as the two upper holes. The tines of the fork are of equal strength (size) and symmetric around the center of the part.

The shock tower is part of an assembly holding a shock absorber and a rocker arm. It has two rod-connection holes of exactly the same size, and two smaller mounting holes. The upper fork is symmetric and is built to hold a shock absorber allowing a single axis of motion. It contains two weight-reducing pockets centered on the main body of the part.

The upright forms the basic foundation of the wheel assembly of the car. It mounts the brake calipers for the wheel using holes on the periphery flanges. It has two weight reducing pockets that are centered in the part, have parallel walls, and are concentric on the center hole, which accepts the wheel axle. All the mounting holes accept the same size screws.

In each case, the designs for the parts are natural and efficient, devised to provide strength and low weight, symmetry, and ease of assembly and manufacturing. As will be shown, the constrained optimization process has in each case produced a new model that captures these intents, thus increasing the overall accuracy of the model while enforcing the criteria of the original design.

6.3 The Process

The reverse engineering process was described in the introductory and background chapters of this dissertation. The processes necessary for recreating models are summarized here:

1. Data Acquisition and Registration
2. Initial Segmentation and Plane Fitting
3. Initial Feature-by-Feature Estimation
4. Automatic Constraint Hypothesis
5. DOF Reductions and Boundary Function Creation
6. Constrained Optimization

The remainder of this chapter discusses each phase and the results associated with it. The important research addressed by this work concerns stages 4 through 6. Additional work was invested in phases 2 and 3 to rapidly produce initial data for the optimization phases.

6.4 Data Acquisition

Each exemplar part was scanned multiple times by a Digibot II laser scanner (See Section 2.8) in different (approximately orthogonal) orientations to achieve a complete covering of the entire object. Refixturing was necessary because not all surfaces could be captured with only one view. The data from each scan were merged into one data cloud by calculating equivalent planar surfaces in each view of the data and optimizing a transformation (rotational and translational) to minimize the distance between these planes.

Each part was sprayed with a crack developer solution that spreads a thin layer of white powder across the part. This powder was measured to average approximately 25 microns in thickness on a planar surface.

For small hole features, screws or plugs were inserted in order to achieve more accurate data for these areas. Because the plugs were aligned with the bounding plane of the feature, it was still possible to project the plug data into the bounding plane to analyze the center of the hole.[28]

6.5 Initial Segmentation and Plane Fitting

The exemplar parts utilized in this work are in the class of 2.5D geometry, explicitly, 2D contours that have been extruded along the Z-axis with occasional minor off axis machining operations.[29] Each plane used in the manufacture of the part forms an "Anchor" for the extrusion of the 2D feature into 3D. Knowledge of this class of parts allows for intelligent segmentation of the data cloud representing the object. First, a stochastic plane finding method is run over the data, automatically classifying sections of data corresponding to the main planar regions of the part. This rapidly reduces the total number of points in the data cloud, which in turn speeds the collection of smaller planes and eventually the segmentation of the features themselves.

After all the points associated with each anchor plane have been defined, an optimization process is invoked which optimizes over all aligned planes. The planes are represented by a common normal for the plane group and the minimum distance from each plane to the origin, thus requiring an N+3 DOF parameter vector, where N is equal to the number of planes. The error minimized is the RMS distance from each data point to its corresponding plane. A simplex search is used to compute the optimization.

6.6 Initial Feature by Feature Estimation

Once the plane segmentation has been accomplished, the remaining data represent the various feature regions (data associated with the inherent features of the part). This

[28] This method of hole sensing combined with the inherent nature of feature-based modeling allows the center of the hole to be fit accurately and the desired radius to be inserted by the engineer. For the purposes of the errors reported in this chapter, no values were inserted by hand. All radii were calculated from the sensed data.

[29] Off axis machining is accomplished by refixturing the object and re-machining. It should be noted that fixturing errors can often be a significant source of manufacturing error.

leaves each region spatially segregated from its neighbors and thus more easily pulled out by semi-automatic means.

At this point, the user selects a feature (which includes specifying the bounding planes and a data point in the data cloud which is part of the feature), and the data associated with that feature is automatically segmented from the data cloud. The user can interactively clip any elements from the segmentation that is deemed outliers. The resulting data is projected into the specified anchor plane, forming a list of 2D points representing the feature outline. A contour finding method is then applied to construct an arc and line decomposition of the data. Each line and arc is fit separately producing an initial description of the feature.

Because of varying levels of noise and scale across a part, the contour routine can mistakenly label an arc segment as a line segment. The user is allowed to interactively correct the resulting geometry before accepting the final contour and passing it on to the next stage. Once the feature has been defined, all data points associated with the feature is classified and saved for use in the optimization process. This makes each subsequent feature segmentation easier in the sense that fewer outliers and misclassifications are likely. These steps are repeated until the entire data cloud has been classified.

The result of this phase of the process is a feature-by-feature decomposition of the part along with initial geometry approximations and associated segmentations. The geometry is then used to hypothesize the features and the data points are used during the optimization process.

6.7 Automatic Constraint Acquisition

Engineers are good at implicitly visualizing possible constraints given an exemplar part, yet this process can be tedious. Stating, line 1 is parallel to line 2, and line 3 is perpendicular to line 2, and line 4 is parallel to line1, etc., for a large number of features is quite repetitive and could result in the user unintentionally missing constraints. This research has shown that the constraint identification and assertion process is well suited to automation. Table 6-1 displays the number of constraints identified on the geometry of each exemplar. In all cases the majority of constraints were accurately hypothesized. The missing constraints were the result of comparing the parametric values of small

Table 6-1: Constraint Hypotheses

Exemplar Part	***Hypothesized Constraints***	***Incorrect Constraints***	***Missed Constraints***
Steering Arm	3	0	0
Shock Plate	22	0	2
Lower Link	41	2	0
Shock Tower	20	0	1
Upright	79	8	0

primitives that were not accurately fit by the initial primitive fitting phase due to noisy data and incorrect segmentations.

The incorrect constraints found for the Lower Link involved a concentric arc constraint between the outer profile and the inner pocket contour. The designer chose slightly different rounding radii for each arc. This is a case where because of the small nature of the primitive, the arc centers overlapped enough to hypothesize their concentricity. Had the constraint been allowed through to the optimization process, the inner and outer profile arcs would have been forced into alignment. The resulting geometrical differences would have not been visible to the engineer, nor would the effect have been adverse.

The outer profile of the upright consists of four flanges that are similar to boss features surrounding the four mounting holes. For each of these flanges, it was hypothesized that they were actually boss features, when in fact, the bounding arcs are offset by small line segments and do not truly center on the hole. When optimized with these constraints, spikes in the error were noted for the flanges as the two outer arcs were forced into a single arc. From this information it was possible for the engineer to reevaluate the original model and discover the need for these line segments.

The constraint hypothesizer accurately identified the vast majority of constraints used in the design of the exemplar parts based on the initial parametric and geometric values fit to the data. This technique has proven robust and effective. Nevertheless, the routine is sensitive to the fitting of the primitives in the object and would need to be adjusted to compensate for poorer data sources or objects of larger scales. Thus it is important to allow an interactive environment where the engineer can override the hypothesized constraints or assert new constraints.

A final note should be made about the complexity of the constraint hypothesizer. As implemented, the hypothesizer is doing N-squared comparisons, where N represents the number of primitives. Many of these checks result in a simple comparison that shows no possible appropriate constraints between the primitives. Of those primitives that have possible correspondences, a simple mathematical comparison is all that is required. The largest model in the exemplar set (the Upright) contains approximately 90 primitives. The time required to analyze this model was 0.1 seconds.[30] For extremely large and complicated parts or assemblies, it may be necessary to optimize the algorithm. Certainly as the DOFs of the object are reduced based on initial constraints, the remaining number of checks could be reduced.

6.8 DOF Reduction and Boundary Function Creation

Chapter 4 provides a description of the DOF reduction methods and the creation of boundary functions. The initial DOFs for each model, the reduced number, and the total number of boundary functions applied to each reverse engineering process are presented below. As can be seen in Table 6-2, a significant amount of simplification between a full geometric description and a constrained constructive representation can be achieved via the DOF reduction strategy. The first column represents the number of DOFs to geometrically build the object. The second column reports the number used after the domain specific primitives have been applied. The third column represents the number of DOFs after the global constraints were applied. This is the size of the feature vector during the constrained optimization. The final column shows the number of constraints that were not applicable to symbolic substitution methods and required a penalty function implementation

[30] Pentium III, 1.0 Gigahertz Processor

Table 6-2: Degree of Freedom Reductions and Penalty Functions

Exemplar Part	Initial DOFs	With Domain Specific Primitives and Pragmatics	With Functional Constraints	Number of Penalty Functions
Steering Arm	31	18	15	1
Shock Plate	132	48	20	9
Lower Link	144	72	26	5
Shock Tower	92	45	22	3
Upright	264	144	68	7

.

The large amount of simplification shown here directly leads to the increased accuracy found in the next section. It also is indicative of the well-defined nature of this domain of parts, ratifying the decision to use domain specific knowledge.

6.9 Error Analysis

The error analysis of the reverse engineering process comes in several flavors. In the past, model creation techniques have described their success in terms of how well the model fits the sensed data. Thompson et al. [36] describe various methods of determining error between reconstructed models. These include positional and local surface shape error, corresponding to parametric and volumetric errors metrics.

Table 6-3 lists the RMS values calculated on a feature-by-feature basis for the exemplar parts. The values are in microns and represent the positional error between the reverse engineered model and the original model. The second column represents the model fit using low-level constraints (pragmatics and features). The third column represents the model constructed via the constrained optimization process with as much knowledge as possible applied. In almost all cases the RMS errors between the recreated features and the corresponding feature on the original model have been reduced by the use of constraints. In the few instances where the error has increased, the optimization has pulled one feature away from its true position in order to optimize a separate feature.

Table 6-3: RMS Error on Exemplar Parts in Microns

Exemplar Part and Features	Feature-based model	Constrained Model
Steering Arm		
Tie Rod Hole	90	49
Mounting Hole	89	62
Profile	39	39
Shock Plate		
H1	41	32
H2	63	23
H3	70	27
Profile	25	20
Pocket 1	186	49
Pocket 2	192	49
Lower link		
Pocket 1	264	131
Pocket 2	157	131
Profile	29	24
Shock Tower		
Profile	58	41
Pocket 1	166	121
Pocket 2	148	121
Hole 1	77	53
Hole 2	29	47
Mounting Hole 1	74	35
Mounting Hole 2	64	48

Table 6-3: continued

Exemplar Part and Features	Feature-based model	Constrained Model
Upright		
Flange Hole 1	523	259
Flange Hole 2	446	263
Flange Hole 3	360	366
Flange Hole 4	140	278
Profile 1	57	41
Profile 2	50	44
Profile 3	61	67
Pocket 1	71	37
Pocket 2	63	39
Boss	22	28
Big hole	35	28

In all exemplar parts, the design intent behind the original model has been effectively captured. The optimization process has forced the constrained model into feasibility, reducing parametric differences to zero. Table 6-4 represents the parametric discrepancies between associated features of the Shock Tower as reverse engineered with local (purely feature-based) and global constraints. The feature-based model contains wide variations depending on the noise associated with each feature. From 0 to 203 microns of discrepancy can be seen. To attempt to convey the benefit of the constrained methodology, several additional examples are presented below.

Consider the basic design of the Steering Arm. The most prevalent advantage of this method on the Steering Arm is the concentric placement of the tie rod hole inside the outer profile. The calculated center of the hole using the constrained method was exactly concentric with the outer profile, while using a local, feature by feature fit, the center was 101 microns off the true center.

The other significant result is the centering of the mounting hole based on the exterior profile. Additionally, the line segments on the outer profile were aligned perpendicular and parallel to their neighbors. In the feature-based method, the normal of the line segments were off by .08 and .06 degrees from their designed value. These small

Table 6-4: Parametric Differences in Microns

Shock Tower Feature Decomposition	Without Constraints	With Constraints
Difference between the radii of hole 1 and 2	76	0
Difference between the radii of the mounting hole 1 and 2	0	0
Pocket 1 and 2 width difference	215	0
Pocket 1 and 2 offsets	50-100	0
Width of the webbing between pocket 1 and the outer profile	61	0
Offset of the centers of the mounting holes 1 and 2	50, 200	0
Offset of the cube hole center from the exterior profile arc center	25	0

discrepancies are not necessarily important geometrically, but parametrically remove the nice properties associated with true parallelism and perpendicularity.

It should be noted that the outer profiles of a part are sensed more accurately than interior features because fewer occlusions occur and the profile can be aligned normal to the laser. This can lead to positional accuracy on par to models created using functional constraints. Nevertheless, functionally constrained geometry is superior as shown in Figure 6-4 representing a highly magnified look at the three profiles that make up the exterior of the Upright. The three profiles represent the rectangular section near the top of the part, the main contour section of the part without the flanges, and the main contour section with the flanges. The fits of the three exterior profiles made in isolation to each other vary up to 50 microns and are not aligned upon one another as would be required for constructing a solid model from the features or when considering machining a new part. The profiles that were generated under the constrained optimization align on each other as expected. Thus the necessity of optimizing based on global constraints can clearly be seen.

6.10 Summary

The results presented here show significant increases in the geometric accuracy of the resulting reverse-engineered models while enforcing dependencies among the various inter-related geometries of each part similar to those found in the original designs. Thus the new models are both geometrically closer to the original design, and parametrically more in keeping with the design intent behind the parts. Finally, the ability to automatically hypothesized constraints has been shown to be effective, and the ability to construct model representations with far fewer DOFs, based on asserted constraints, has been demonstrated.

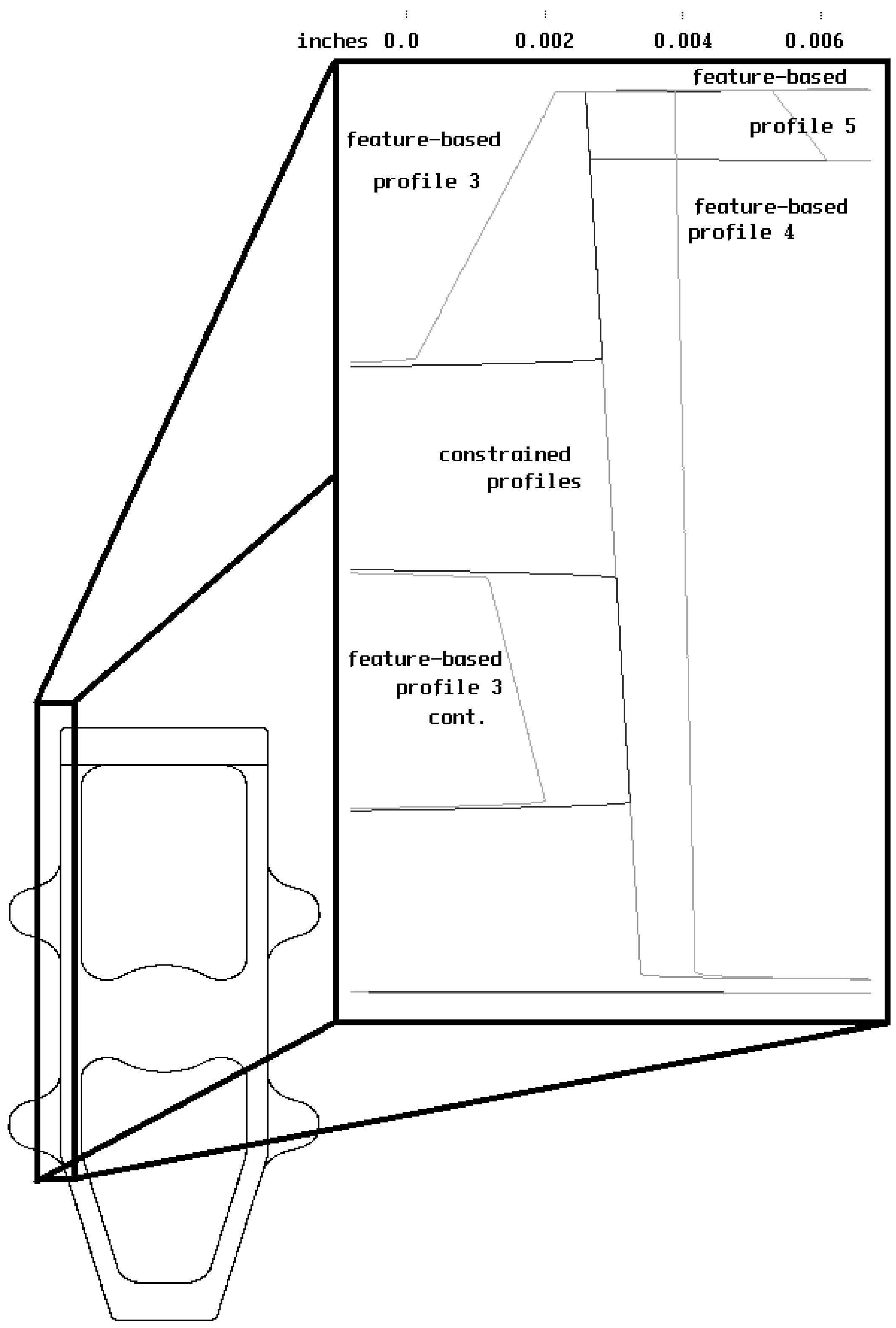


Figure 6-4: Upright Exploded View

CHAPTER 7

CONCLUSION

The field of reverse engineering requires model creation techniques that capture the high tolerances and design attributes necessary to successfully machine replacement parts. Current methods of reverse engineering usually require the manual inspection and measurement by an engineer or use generic model fitting techniques based on the data acquired from general purpose sensors. The physical inspection of a part can be a time-consuming process requiring manufacturing and design expertise, often resulting in approximations where hand held calipers cannot easily describe the geometry of the part. Generic reverse engineering strategies are usually not appropriate to the domain of manufactured parts.

Successful models of manufactured parts demand high precision modeling that ideally captures the functional properties of the part in addition to the geometry. Such models must be amenable to manufacture and support re-engineering. Appropriate models for manufacture often require a precision greater than that directly available from the scanned data. Such data are usually poor because of the reflectance property of the parts and the fact that the most difficult areas to sense are often those where the highest tolerances are needed, such as the discontinuities between features. Previous reverse engineering systems were not developed to handle the data and requirements of this domain and therefore result in poorer accuracy and less useful representations.

In the reverse engineering of mechanical 2.5D parts, it must be recognized that these parts are, by their very nature, artifacts designed with specific properties that can be leveraged in the model recreation process. The geometry as designed is limited by considerations such as effectiveness, functionality, simplicity, and manufacturability. This design and manufacturing knowledge can be codified via the use of constraints to guide the optimization process, effectively increasing the accuracy and appropriateness of the final model. Appropriate models can be created by applying domain specific primitives, pragmatics, and functional constraints, inside an optimization framework, despite the error associated with the sensing process.

Constraint-based reverse engineering attempts to recreate a model of a part from an exemplar object that is faithful to the original design and captures the geometry and function of the part with greater accuracy than previously attainable. The resultant models logically depict the objects as they are likely to have been designed and contain dependencies that are useful if any redesign is necessary. In the presence of noisy data, the parametric optimization minimizes the positional error associated with each feature of the part while constraints maintain the functional properties of the features as predicted by manufacturing and design knowledge.

7.1 Contributions

Reverse engineering should not be considered simply a "fitting" process based on sensor data, but should instead be couched as an optimization process constrained by the sensor data and by knowledge of design artifacts. The application of constraints (based on domain specific knowledge) provides a method for reverse engineering more accurate and useful models.

This work has identified a set of constraints that encapsulate the design of a representative set of 2.5D manufactured parts. These constraints have been formulated as symbolic manipulations of the algebraic representation of the model and also (when symbolic transformations are not possible) as penalty functions based on the parametric and geometric representations of the model. Further, this work has demonstrated that likely constraints can be automatically hypothesized.

The domain of 2.5D manufactured parts requires high accuracy models, which are appropriate for redesign or machining. Current data acquisition methods for use with these types of objects are not capable of recreating a desirable model appropriate to the domain. This dissertation has shown that knowledge can be applied to the model recreation process in the forms of geometric constraints. The results presented for the exemplar parts show not only geometric accuracy in terms of lower RMS error values, but also an improved parametric representation. The ability to exploit domain specific representations, pragmatics, and global constraints through the use of optimization algorithms is a significant contribution toward the area of high tolerance model recreation.

7.2 Systems

An interactive system was created for visually displaying point cloud data and rapidly segmenting and fitting the data cloud resulting in an initial feature-based model. A separate program was created to analyze this model, hypothesizing constraints based on design and manufacturing knowledge. Finally an optimization framework was created which accepts the initial feature-based model, the list of constraints, and the data points, and uses generic optimization routines to construct the optimized model. This model is directly transferable to a feature-based CAD package for manufacture or redesign.

7.3 Future Work

While constraint optimization is a powerful method for reverse engineering, there are several issues of concern with the current system and numerous areas of research available in the field of reverse engineering itself.

The initial model creation process is still far from automatic. Advances could be made in the area of automatic segmentation. Much as the automatic plane finder is an initial step, techniques for identifying self contained "feature clouds" between planes could be made. These segmentations could be automatically fit, both as holes and contours, and an error metric could decide if either effectively captures the geometry of the cloud.

In the area of optimization, the weights associated with features during the evaluation of the objective function could be hypothesized based on knowledge of the sensor. Additionally, multiple data sources could be integrated into the system, perhaps adding an interactive step to the sensing process where features and constraints guide the sensing process during the model creation. Further, the model as developed could be used to drive a CMM that would check and improve the accuracy of the model.

Another area of interest is the adaptation of this work toward a larger set of possible geometric forms, including identifying and implementing constraints on manufactured parts created using five axis machining. Other interesting domains could also be addresses, such as parts created on lathes, by castings, or by other manufacturing techniques. Additional 2.5D constraints, such as radial symmetry, should also be implemented to extend the usefulness of the current system

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