Outlier Robust ICP for Minimizing Fractional RMSD

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Introduction

Aligning an input data set to a model data set is fundamental to many important problems such as scanned model reconstruction, structural biochemistry, and medical imaging. The input data and the model data are typically given as a set of points with the relative positions not known, making the task of registering them nontrivial.

A popular approach to solving this problem is known as the iterative closest point (ICP) algorithm [BM92, CM92] which alternates between finding the optimal correspondence between points, and finding the optimal transformation of one point set onto the other. As both steps reduce the distance between the point sets, this process converges, but only to a local minimum.

However, ICP, and its many variations, are vulnerable to point sets with outliers. Because ICP will find correspondences for all points, and then find the optimal transformation for the entire point set, the outliers will skew the alignment. Many heuristics have been suggested [DF02, CSK05] including only aligning points within a set threshold, but most of these techniques are not guaranteed to converge, and thus can possibly go into an infinite loop, or require an expensive check to prevent this. If the fraction f of points which are outliers is known, then Trimmed ICP (TrICP) [CSK05] can be used to find the optimal alignment of the most relevant fraction f of points. However, this fraction is rarely known a priori. If an alignment is given then RANSAC-type methods can be used to determine a good threshold for determining these outliers. None of these existing approaches both find a local minimum and converge.

Our contributions. Our solution to these problems is to incorporate the fraction of points which are outliers into the function being optimized. To this end, this abstract (see full version [PLT06] for details) makes the following contributions:

- We formalize a new distance measure between point sets which accounts for outliers: FRMSD.
- We provide an algorithm, Fractional ICP, to optimize FRMSD which we prove to converge to a local optimum in the correspondence, transformation, and fraction of outliers.
- We show that Fractional ICP finds an alignment with only the points which are more likely to be inliers than outliers.

Fractional RMS Distance

Consider two point sets $D, M \in \mathbb{R}^d$. The goal of this paper is to align an input data set D to a model data set M under some class of transformations, $\mathcal{T}$, such as rigid motions. We assume M and D are quite similar and there exists a strong correspondence between most points in the data; however, there may be outliers, points in either set which are not close to any point in the other set. Our goal is to define and minimize over a set of transformations a relevant distance between these two point sets.

We define the root mean squared distance

$$\text{RMSD}(D, M, T, \mu) = \sqrt{\frac{1}{|D|} \sum_{p \in D} ||T(p) - \mu(p)||^2},$$

and we seek to minimize this quantity over a set of transformations $T \in \mathcal{T}$ and matchings $\mu : D \to M$. RMSD is quite susceptible to outliers because the squared distance gives a large weight to outliers. To counteract this, for a fraction $f \in [0, 1]$ choose the $f|D|$ points with the smallest residual distance $r = ||p - \mu(p)||$. Let D_f be this set.

We define fractional root mean squared distance

$$\text{FRMSD}(D, M, f, T, \mu) = \frac{1}{f^\lambda} \sqrt{\frac{1}{|D_f|} \sum_{p \in D_f} ||T(p) - \mu(p)||^2},$$

and we seek to minimize this quantity over a set of transformation $T \in \mathcal{T}$, matchings $\mu : D \to M$, and fractions $f \in [0,1]$.

Value of λ. Under reasonable assumptions on the distribution of outliers and through some straightforward but tedious probability theory we can show that for $\lambda = 1.3$ (resp. $\lambda = 0.95$) for point sets in $\mathbb{R}^2$ (resp. $\mathbb{R}^3$), that FRMSD considers only the points which are more likely to be inliers than to be outliers. This is somewhat dependent on the noise among the inliers, but only weakly dependent on the fraction of inliers. We show empirically that as λ is increased up to 4 or 5, the minimizing fraction f and the RMSD value does not change much. Thus, the algorithm is not sensitive to this regularization parameter.

Fractional ICP

A simple modification of ICP, shown in Algorithm 0.1, will find a local minimum of FRMSD. We refer to this algorithm as Fractional ICP or FICP.

Algorithm 0.1 FICP(D, M)

1: Compute $\mu_0 = \arg \min_{\mu_0 : D \to M} \text{RMSD}(D, M, T_0, \mu_0)$.
2: Compute $\arg\min_{f_0 \in [0,1]} \text{FRMSD}(D, M, f_0, T_0, \mu_0)$.
3: $i \leftarrow 0$.
4: **repeat**
5: $\quad i \leftarrow i + 1$.
6: $\quad$ Compute $\arg\min_{D_f} \text{RMSD}(D_f, M, T_{i-1}, \mu_{i-1})$.
7: $\quad$ Compute $\arg\min_{T_i \in \mathcal{T}} \text{RMSD}(D_f, M, T_i, \mu_{i-1})$.
8: $\quad$ Compute $\arg\min_{\mu_i : D \to M} \text{RMSD}(D, M, T_i, \mu_i)$.
9: $\quad$ Compute $\arg\min_{f_i \in [0,1]} \text{FRMSD}(D, M, f_i, T_i, \mu_i)$.
10: **until** $(u_i = u_{i-1}$ and $f_i = f_{i-1})$

Implementation. Given a standard implementation of ICP, we need the additional steps of computing the subset D_f (step 6) and computing the fraction f (step 9). Since the rest of the algorithm is unchanged, most variations of ICP can incorporate this adaptation. The subset D_f can be computed by sorting the residuals $r = ||p - \mu(p)||$ and letting D_f be the $f|D|$ points with smallest corresponding residuals. Once the residuals are sorted, the fraction f can be computed by considering all $|D|$ possible subsets and choosing the one with smallest value in FRMSD.

Convergence of algorithm. Algorithm 0.1 converges to a local minimum of FRMSD(D, M, f) over the space of all transformations, matchings, and fractions of points used in the matching. This is a local minimum in a sense that if all but one of transformations, matchings, or fractions is fixed, then the value of the remaining variable cannot be changed to decrease the value of FRMSD(D, M, f).

Theorem 0.1 For any two points sets $D, M \in \mathbb{R}^d$, Algorithm 0.1 converges to a local minimum of FRMSD(D, M, f, T, μ) over $(f, T, \mu) \in [0,1] \times \mathcal{T} \times \{D \to M\}$.

Experiments

In the full version [PLT06] we demonstrate that FICP has a larger radius of convergence than TrICP, and is faster and more accurate than TrICP and ICP. Also, we empirically demonstrate that FRMSD is not sensitive to λ, but FICP is more robust with λ set larger than its optimal value. Thus we run all experiments, unless otherwise specified, with $\lambda = 3$. After converging, λ can be reset to its optimal value and the process will reconverge quickly.

Figure 1 shows the alignment of the scan at 0° (blue) aligned with the scan at 48° (red) of the Stanford dragon using ICP and FICP. Notice how when the scans are aligned with ICP (right), the points in the dragon's tail are slightly misaligned, whereas with FICP (left), the alignment is much better.

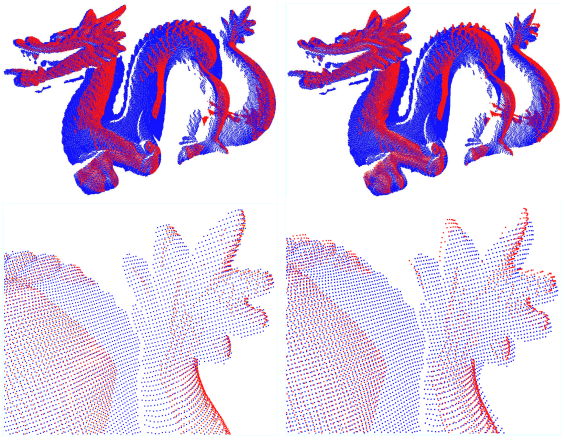

Figure 1: *Alignment of scans for dragon model with ICP (right) and FICP (left). Bottom shows zoomed in view of dragon's tail.*

References

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