# FoDA L3

## Probability Review #2

# Random Variables

$X : \Omega \to \Delta$

$X, Y$

probability density function

$f_X : \Omega \to \mathbb{R}$

$\Pr(X \in A)$

$= \int_{\omega \in A} f_X(X = \omega)\, d\omega$

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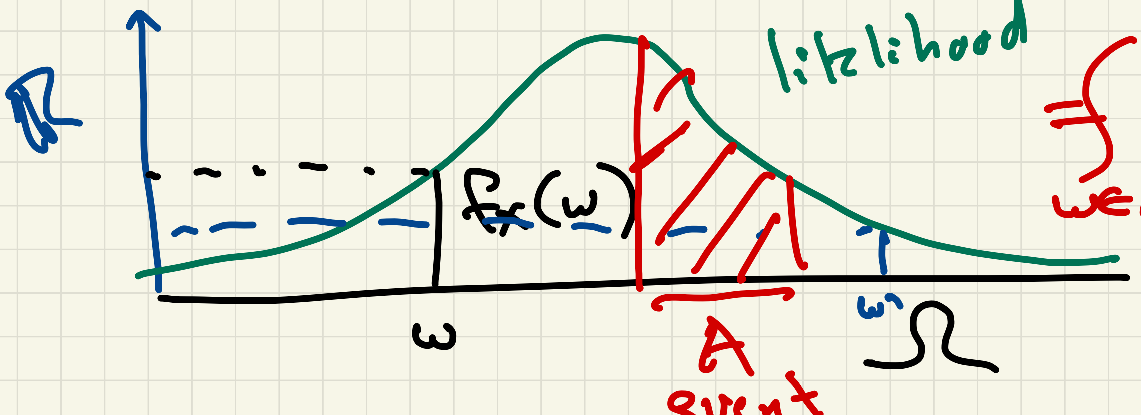


Die random variable $D$

| $\Omega$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $f_D(\omega) = \Pr(D=\omega)$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ |

# Cumulative Density function

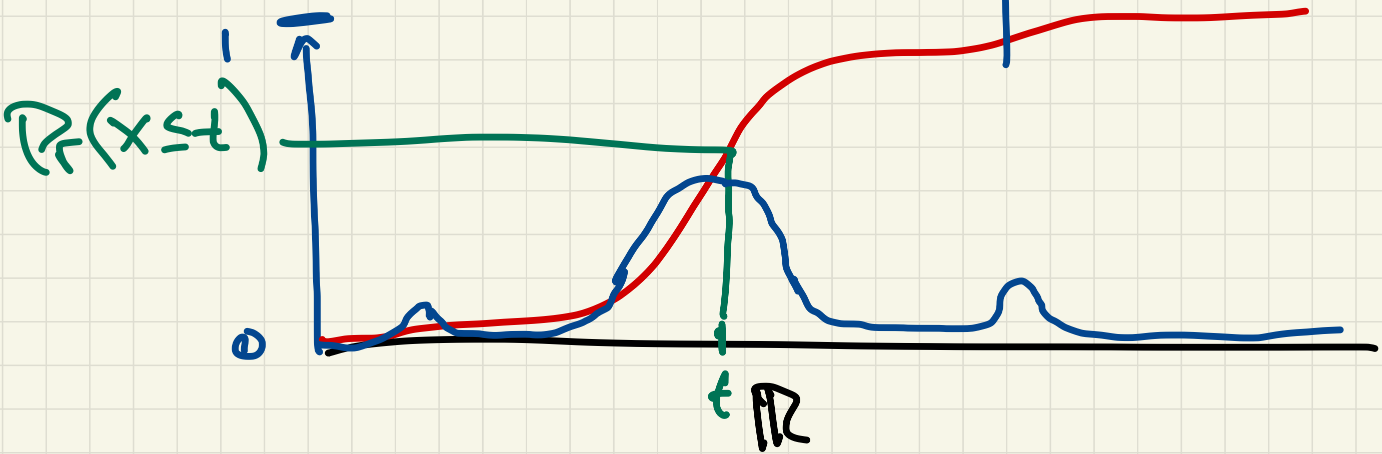
R.V. $X$

pdf $f_X : \cancel{\Omega}\ \mathbb{R} \to \mathbb{R}$

cdf $F_X : \mathbb{R} \to [0,1]$

$$F_X(t) = \int_{x=-\infty}^{t} f_X(x)\,dx$$

$$f_X(t) = \frac{\partial F_X(t)}{\partial t}$$



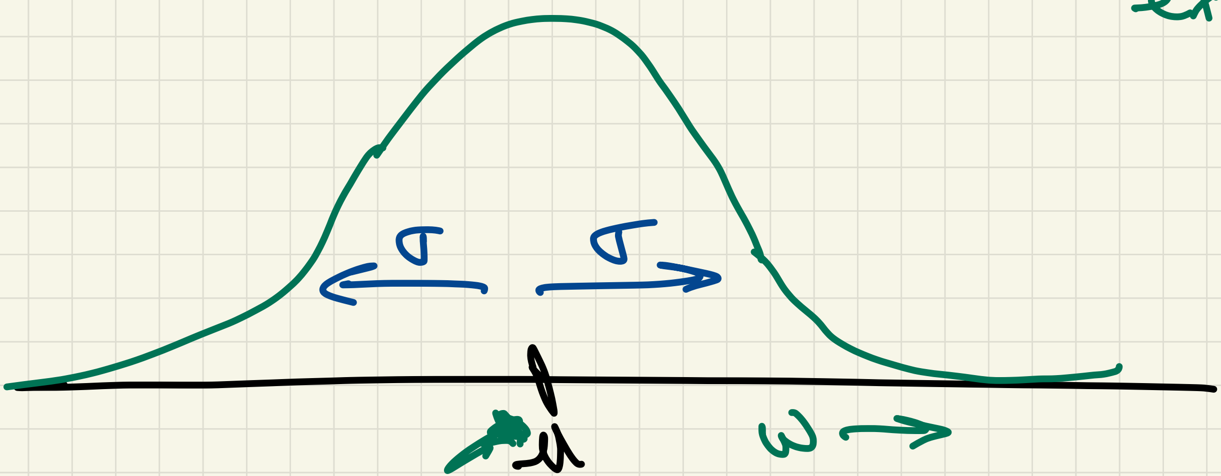
# Normal Random Variable X

pdf $f_X(\omega) = \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left(-\frac{(\omega-\mu)^2}{2\sigma^2}\right)$

$\sigma = 1 \quad \mu = 0$

$\exp(y) = e^y$

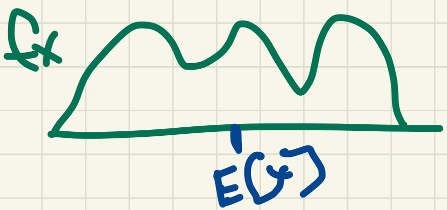
$e = 2.71\ldots$



## Expected Value

R.V. $X$

$$E[X] = \sum_{\omega \in \Omega} \left( \omega \cdot \Pr[X = \omega] \right)$$




$$= \int_{\omega \in \Omega} \omega \cdot f_X(\omega) \, d\omega$$

## Linearity of Expectation

RV $X, Y$ constant $\alpha$

$$E[X + \alpha Y] = E[X] + E[\alpha Y]$$

$$= E[X] + \alpha E[Y]$$

$D : \{1, 2, \dots 6\}$ $\quad$ $Pr[D = j] = 1/6$

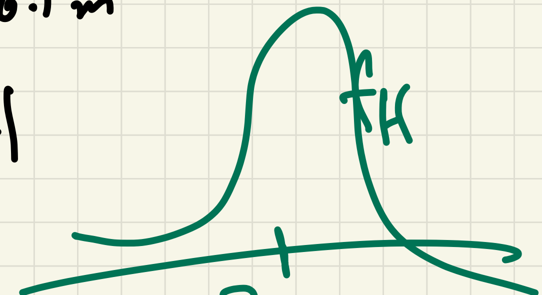
$$E[D] = 1 \cdot \frac{1}{6} + 2 \cdot \frac{1}{6} + 3 \cdot \frac{1}{6} + 4 \cdot \frac{1}{6} + 5 \cdot \frac{1}{6} + 6 \cdot \frac{1}{6}$$

$$= \frac{1}{6}(1 + 2 + \dots 6) = \frac{21}{6} = 3.5$$

# Linearity expectation

heigh $N(\mu, \sigma^2) = H$ $\quad \sigma = 0.1\,m$

$\mu = 1.755\,m$




$E[H] = 1.755\,m$

shoes

| S= | 1cm | 2cm | 3cm | 4cm |
|---|---|---|---|---|
| | 0.1 | 0.1 | 0.5 | 0.3 |

$$E[S] = 1(0.1) + 2(0.1) + 3(0.5) + 4(0.3)\ \text{cm}$$

$$= 3\ \text{cm}$$

$$E[B] = E[100H + S] = E[100H] + E[S] = \begin{array}{l} 100E[H] + E[S] \\ 175.5\text{cm} + 3\text{cm} \end{array}$$

# Variance

standard deviation

$$\sigma_x = \sqrt{Var[x]}$$

$$Var[x] = E\left[(x - E[x])^2\right]$$

$$= E[x^2] - E[x]^2$$

$$E\left[(x-E[x])^2\right] = E\left[x^2 - 2xE[x] + E[x]^2\right]$$

$$= E[x^2] - 2E[x]E[x] + E[x]^2$$

$$= E[x^2] - E[x]^2$$

# Variance

Shoes

| $S =$ | 1 | 2 | 3 | 4 cm |
|---|---|---|---|---|
| | 0.1 | 0.1 | 0.5 | 0.3 |

$$E[S] = 3$$

$$Var(S) = E\Big[\big(S - \overbrace{E[S]}^{3}\big)^2\Big]$$

$$= (0.1)(1-3)^2 + (0.1)(2-3)^2 + (0.5)(3-3)^2 + (0.3)(4-3)^2$$

$$= (0.1)\,4 + (0.1)\,1 + 0 + (0.3)\,1$$

$$= 0.8$$

$$std(S) = \sigma_S = \sqrt{0.8}$$

# Joint, Marginal, Conditional Distribution

R.V. $X, Y$

Co-Variance

$$\mathrm{COV}[X,Y] = E[(X - E[X])(Y - E[Y])]$$

joint pdf $f_{X,Y} : \Omega_X \times \Omega_Y \to \mathbb{R}$

discrete $\quad f_{X,Y}(x,y) = \Pr(X=x,\ Y=y) \in [0,1]$

$$\sum_{x \in X} \sum_{y \in Y} f_{X,Y}(x,y) = 1$$

R.V. $P$ $S$

| | $S$=green | ='red' | =blue |
|---|---|---|---|
| $P$=blue | 0.3 | 0.1 | 0.2 |
| $P$=white | 0.05 | 0.2 | 0.15 |
| $f_S$ | 0.35 | 0.3 | 0.35 |

$$Pr(P=\text{blue} \mid S=\text{red}) = \frac{0.1}{0.3}$$

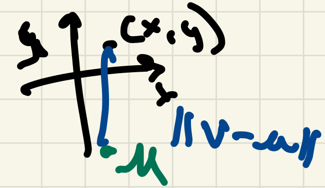
marginal pdf

$$f_S(s) = \sum_{p \in P} f_{S,P}(s,p) = \int_{p \in P} f_{S,P}(s,p)\,dp$$

conditional pdf

$$f_{P|S}(p \mid s=\text{red}) = \frac{f_{P,S}(p, s=\text{red})}{f_S(s=\text{red})}$$

# Gaussian Distribution

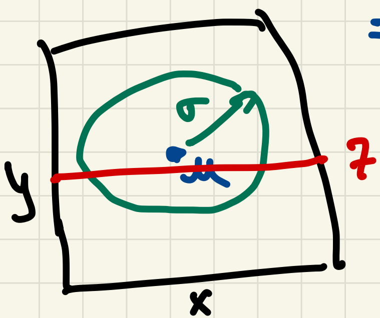



$f_X : \mathbb{R}^2 \to \mathbb{R}$ $\qquad v \in \mathbb{R}^2 = \{(x,y) \in \mathbb{R} \times \mathbb{R}\}$

$v = (v_x, v_y)$

$$f_X(v) = \frac{1}{\sigma^2 \, 2\pi} \exp\left(-\frac{\|v-\mu\|^2}{2\sigma^2}\right)$$

$$= \frac{1}{\sigma^2 \, 2\pi} \exp\left(\frac{-(v_x-\mu_x)^2-(v_y-\mu_y)^2}{2\sigma^2}\right)$$




$f_{X|Y=7}(x \mid y=7)$