FoDA L3

Probability Review #2

pdfs, cdfs, Expectation, Variance,
Joint & Marginal Densities

Aug 30, 2022

Safety and Addressing Sexual Misconduct

The University of Utah values the safety of all campus community members.

Title IX makes it clear that violence and harassment based on sex and gender (which Includes sexual orientation and gender identity/expression) is a civil rights offense subject to the same kinds of accountability and the same kinds of support applied to offenses against other protected categories such as race, national origin, color, religion, age, status as a person with a disability, veterans status or genetic in-formation. If you or someone you know has been harassed or assaulted, you are encouraged to come speak to the School of Computing Advisors and/or to the Title IX Coordinator in the Office of Equal Opportunity and Affirmative Action, 135 Park Building, 801-581-8365, or the Office of the Dean of Students, 270 Union Building, 801-581-7066. For support and confidential consultation, contact the Center for Student Wellness, 426 SSB, 801-581-7776.

To report suspicious activity or to request a courtesy escort, call campus police at 801-585-COPS (801-585-2677). You will receive important emergency alerts and safety messages regarding campus safety via text message. For more information regarding safety and to view available training resources, including helpful videos, visit `https://safeu.utah.edu`.

Random Variables C, T

cancer $C \begin{cases} 1 & \text{yes} \\ 0 & \text{no} \end{cases}$

test $T \begin{cases} 1 & \text{positive} \\ 0 & \text{negative} \end{cases}$

$Pr(C=1, T=1)$

	C=1	C=0
T=1	0.1	0.02
T=0	0.05	0.83

test

$\Rightarrow Pr(T=1) = 0.1 + 0.02 = 0.12$

$P(T=1 \mid C=1) = \dfrac{Pr(C=1 \wedge T=1)}{Pr(C=1)} = \dfrac{0.1}{0.15} = \dfrac{2}{3}$

$Pr(T=1) = 0.12 \neq 0.66$

Density Functions

RV X

$f_X : \Omega \to \mathbb{R}$

Ω: sample space

$\mathbb{R}$: $(0, \infty)$

"likelihood"

probability density function

pdf

$$\int_{\omega \in \Omega} f_X(\omega)\, dx = 1$$

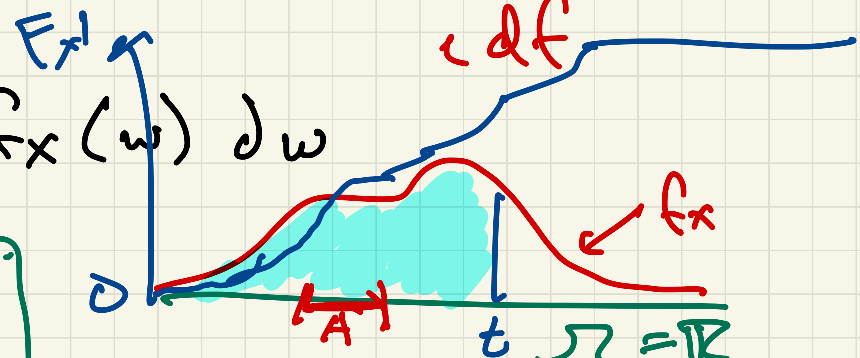
$F_X : \Omega \to [0, 1]$

Ω: $\mathbb{R}$

cumulative density fn

cdf

$$F_X(t) = \int_{\omega=-\infty}^{t} f_X(\omega)\, d\omega$$



Event $A \subset \Omega$

$$\Pr_f[A] = \int_{\omega \in A} f_X(\omega)\, d\omega$$

Expected Value R.V. X

$$E[X] = \sum_{\omega \in \Omega} \left(\omega \cdot \Pr[X=\omega] \right)$$

$$E[X] = \int_{\omega \in \Omega} \omega \cdot f_X(\omega)\, d\omega$$

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discrete X = die $\{1, 2, 3, 4, 5, 6\}$

$$\Pr[X=3] = 1/6$$

$$E[X] = \sum_{\omega \in \{1 \ldots 6\}} \omega \cdot \Pr[X=\omega] = \sum_{\omega} \omega \cdot \frac{1}{6} = \frac{1}{6}(1+2+\ldots+6)$$

$$= \frac{1}{6}(21) = 3.5$$

Linearity of Expectation

2 R.V. X, Y

$$E[X+Y] = E[X] + E[Y]$$

fixed scalar value α

$$E[\alpha X] = \alpha E[X]$$

$$E[X \cdot Y] \neq E[X] \cdot E[Y]$$

height : true height H

: height of shoes S

S =	s = 1	s = 2	s = 3	s = 4	cm
	0.1	0.1	0.5	0.3	

$$(0.1)1 + (0.1)2 + (0.5)3 + (0.3)4 = 3$$

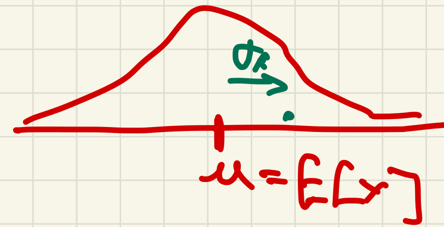
$$H = N(\mu = 1.755 \text{ meters}, \ \sigma = 0.1 m)$$

$$E[H] = 1.755 m$$

$$E[H + S/100] = E[H] + \frac{1}{100}E[S]$$

$$= 1.755 m + \frac{1}{100}(3 cm) = 1.785 m$$

Variance   R.V. X



$$Var[X] = E[(X - E[X])^2]$$

↑ need to calc for Var(X)   ↑ constant

$$= E[X^2] - (E[X])^2$$

$$E[(X - E[X])^2] = E[X^2 + (E[X])^2 - 2X \cdot E[X]]$$

$$= E[X^2] + E[X]^2 - 2E[X] \cdot E[X]$$

$$= E[X^2] - E[X]^2$$

Standard Deviation X

$Var[X]$

$$\text{stdev} = \sigma_X = \sqrt{Var[X]}$$

same units as X or $E[X]$

Covariance X, Y

$$Cov[X, Y] = E\Big[(X - E[X]) \cdot (Y - E[Y])\Big]$$

Joint, Marginal, and Conditional Dist.

R.V. X, Y pdf $f_{X,Y} : \Omega_X \times \Omega_Y \to [0, \infty)$ $\mathbb{R}$

$f_{X,Y}(x, y) \neq$ ~~thing~~ $\Pr(X = x, Y = y)$

$$\int_{x \in \Omega_X} \int_{y \in \Omega_Y} f_{X,Y}(x, y) \, dy \, dx = 1$$