- **Function Abstraction**
- **Type Abstraction**
- **Anonymous Functions**

Big Fish

A function that gets the big fish (> 5 lbs):

```
; big : list-of-nums -> list-of-nums
(define (big l)
  (cond
    [(empty? l) empty]
    [(cons? l)
     (cond
       [(> (first l) 5)
        (cons (first l) (big (rest l)))]
       [else (big (rest l))])]))

(big empty) "should be" empty
(big '(7 4 9)) "should be" '(7 9)
```

Big Fish

Better with **`local`**:

```
; big : list-of-nums -> list-of-nums
(define (big l)
  (cond
    [(empty? l) empty]
    [(cons? l)
     (local [(define big-rest (big (rest l)))]
       (cond
         [(> (first l) 5)
          (cons (first l) big-rest)]
         [else big-rest]))]))
```

Big Fish

Better with **`local`**:

```
; big : list-of-nums -> list-of-nums
(define (big l)
  (cond
    [(empty? l) empty]
    [(cons? l)
     (local [(define big-rest (big (rest l)))]
       (cond
         [(> (first l) 5)
          (cons (first l) big-rest)]
         [else big-rest]))]))
```

Suppose we also need to find huge fish...

Huge Fish

Huge fish (> 10 lbs):

```
; huge : list-of-nums -> list-of-nums
(define (huge l)
  (cond
    [(empty? l) empty]
    [(cons? l)
     (local [(define h-rest (huge (rest l)))]
       (cond
         [(> (first l) 10)
          (cons (first l) h-rest)]
         [else h-rest]))]))
```

Huge Fish

Huge fish (> 10 lbs):

```
; huge : list-of-nums -> list-of-nums
(define (huge l)
  (cond
    [(empty? l) empty]
    [(cons? l)
     (local [(define h-rest (huge (rest l)))]
       (cond
         [(> (first l) 10)
          (cons (first l) h-rest)]
         [else h-rest]))]))
```

How do you suppose I made this slide?

Huge Fish

Huge fish (> 10 lbs):

```
; huge : list-of-nums -> list-of-nums
(define (huge l)
  (cond
    [(empty? l) empty]
    [(cons? l)
     (local [(define h-rest (huge (rest l)))]
       (cond
         [(> (first l) 10)
          (cons (first l) h-rest)]
         [else h-rest]))]))
```

How do you suppose I made this slide?

Cut and Paste!

The Trouble With Cut and Paste

```
; big : list-of-nums -> list-of-nums
(define (big l)
  (cond
    [(empty? l) empty]
    [(cons? l)
     (cond
       [(> (first l) 5)
        (cons (first l) (big (rest l)))]
       [else (big (rest l))])]))
```

cut and paste

```
; huge : list-of-nums -> list-of-nums
(define (huge l)
  (cond
    [(empty? l) empty]
    [(cons? l)
     (cond
       [(> (first l) 10)
        (cons (first l) (huge (rest l)))]
       [else (huge (rest l))])]))
```

The Trouble With Cut and Paste

```
; big : list-of-nums -> list-of-nums
(define (big l)
  (cond
    [(empty? l) empty]
    [(cons? l)
     (cond
       [(> (first l) 5)
        (cons (first l) (big (rest l)))]
       [else (big (rest l))])]))
```

cut and paste

```
; huge : list-of-nums -> list-of-nums
(define (huge l)
  (cond
    [(empty? l) empty]
    [(cons? l)
     (cond
       [(> (first l) 10)
        (cons (first l) (huge (rest l)))]
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The Trouble With Cut and Paste

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; big : list-of-nums -> list-of-nums
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  (cond
    [(empty? l) empty]
    [(cons? l)
     (cond
       [(> (first l) 5)
        (cons (first l) (big (rest l)))]
       [else (big (rest l))])]))
```

cut and paste

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; huge : list-of-nums -> list-of-nums
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  (cond
    [(empty? l) empty]
    [(cons? l)
     (cond
       [(> (first l) 10)
        (cons (first l) (huge (rest l)))]
       [else (huge (rest l))])]))
```

After cut-and-paste, improvement is twice as hard

The Trouble With Cut and Paste

```
; big : list-of-nums -> list-of-nums
(define (big l)
  (cond
    [(empty? l) empty]
    [(cons? l)
     (local [(define big-rest (big (rest l)))]
       (cond
         [(> (first l) 5)
          (cons (first l) big-rest)]
         [else big-rest]))]))
```

cut and paste

```
; huge : list-of-nums -> list-of-nums
(define (huge l)
  (cond
    [(empty? l) empty]
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     (local [(define h-rest (huge (rest l)))]
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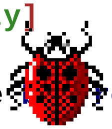
         [(> (first l) 10)
          (cons (first l) h-rest)]
         [else h-rest]))]))
```

The Trouble With Cut and Paste

```
; big : list-of-nums -> list-of-nums
(define (big l)
  (cond
    [(empty? l) empty]
    [(cons? l)
     (local [(define big-rest (big (rest l)))]
       (cond
         [(> (first l) 5)
          (cons (first l) big-rest)]
         [else big-rest]))]))
```



cut and paste

```
; huge : list-of-nums -> list-of-nums
(define (huge l)
  (cond
    [(empty? l) empty]
    [(cons? l)
     (local [(define h-rest (huge (rest l)))]
       (cond
         [(> (first l) 10)
          (cons (first l) h-rest)]
         [else h-rest]))]))
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The Trouble With Cut and Paste

```
; big : list-of-nums -> list-of-nums
(define (big l)
  (cond
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    [(cons? l)
     (local [(define (big-rest (big (rest l)))]
       (cond
         [(> (first l) 5)
          (cons (first l) big-rest)]
         [else big-rest]))]))
```
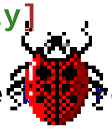

cut and paste

```
; huge : list-of-nums -> list-of-nums
(define (huge l)
  (cond
    [(empty? l) empty]
    [(cons? l)
     (local [(define (h-rest (huge (rest l)))]
       (cond
         [(> (first l) 10)
          (cons (first l) h-rest)]
         [else h-rest]))]))
```

After cut-and-paste, bugs multiply

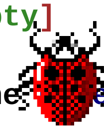
The Trouble With Cut and Paste



```
; big : list-of-nums -> list-of-nums
(define (big l)
  (cond
    [(empty? l) empty]
    [(cons? l)
     (local [(define big-rest (big (rest l)))]
       (cond
         [(> (first l) 5)
          (cons (first l) big-rest)]
         [else big-rest]))]))
```

cut and paste

Avoid cut and paste!



```
; huge : list-of-nums -> list-of-nums
(define (huge l)
  (cond
    [(empty? l) empty]
    [(cons? l)
     (local [(define h-rest (huge (rest l)))]
       (cond
         [(> (first l) 10)
          (cons (first l) h-rest)]
         [else h-rest]))]))
```

After cut-and-paste, bugs multiply

How to Avoid Cut-and-Paste

Start with the original function...

```
; big : list-of-nums -> list-of-nums
(define (big l)
  (cond
    [(empty? l) empty]
    [(cons? l)
     (local [(define big-rest (big (rest l)))]
       (cond
         [(> (first l) 5)
          (cons (first l) big-rest)]
         [else big-rest]))]))
```

How to Avoid Cut-and-Paste

... and add arguments for parts that should change

```
; bigger : list-of-nums num -> list-of-nums
(define (bigger l n)
  (cond
    [(empty? l) empty]
    [(cons? l)
     (local [(define r (bigger (rest l) n))]
       (cond
         [(> (first l) n)
          (cons (first l) r)]
         [else r]))]))
```

How to Avoid Cut-and-Paste

... and add arguments for parts that should change

```
; bigger : list-of-nums num -> list-of-nums
(define (bigger l n)
  (cond
    [(empty? l) empty]
    [(cons? l)
     (local [(define r (bigger (rest l) n))]
       (cond
         [(> (first l) n)
          (cons (first l) r)]
         [else r]))]))

(define (big l) (bigger l 5))

(define (huge l) (bigger l 10))
```

Small Fish

Now we want the small fish:

Small Fish

Now we want the small fish:

```
; smaller : list-of-nums num -> list-of-nums
(define (smaller l n)
  (cond
    [(empty? l) empty]
    [(cons? l)
     (local [(define r (smaller (rest l) n))]
       (cond
         [(< (first l) n)
          (cons (first l) r)]
         [else r]))]))

(define (small l) (smaller l 5))
```

Small Fish

Now we want the small fish:

```
; smaller : list-of-nums num -> list-of-nums
(define (smaller l n)
  (cond
    [(empty? l) empty]
    [(cons? l)
     (local [(define r (smaller (rest l) n))]
       (cond
         [(< (first l) n)
          (cons (first l) r)]
         [else r]))]))

(define (small l) (smaller l 5))
```

Don't cut and paste!

Sized Fish

```
; sized : list-of-nums num ... -> list-of-nums
(define (sized l n COMP)
  (cond
    [(empty? l) empty]
    [(cons? l)
     (local [(define r
               (sized (rest l) n COMP))]
       (cond
         [(COMP (first l) n)
          (cons (first l) r)]
         [else r]))]))

(define (bigger l n) (sized l n >))
(define (smaller l n) (sized l n <))
```

Sized Fish

```
; sized : list-of-nums num ... -> list-of-nums
(define (sized l n COMP)
  (cond
    [(empty? l) empty]
    [(cons? l)
     (local [(define r
                (sized (rest l) n COMP))]
       (cond
         [(COMP (first l) n)
          (cons (first l) r)]
         [else r]))]))

(define (bigger l n) (sized l n >))
(define (smaller l n) (sized l n <))
```

Does this work? What is the contract for **sized**?

Functions as Values

The definition

```
(define (bigger l n) (sized l n >))
```

works because *functions are values*

Functions as Values

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```

works because *functions are values*

- **`10`** is a **`num`**
- **`false`** is a **`bool`**

Functions as Values

The definition

```
(define (bigger l n) (sized l n >))
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works because *functions are values*

- `10` is a `num`
- `false` is a `bool`
- `<` is a `(num num -> bool)`

Functions as Values

The definition

```
(define (bigger l n) (sized l n >))
```

works because *functions are values*

- **`10`** is a **`num`**
- **`false`** is a **`bool`**
- **`<`** is a **`(num num -> bool)`**

So the contract for **`sized`** is

```
; list-of-nums num (num num -> bool)
; -> list-of-nums
```

Sized Fish

```
; sized : list-of-nums num (num num -> bool)
; -> list-of-nums
(define (sized l n COMP)
  (cond
    [(empty? l) empty]
    [(cons? l)
     (local [(define r
               (sized (rest l) n COMP))]
       (cond
         [(COMP (first l) n)
          (cons (first l) r)]
         [else r]))]))

(define (tiny l) (sized l 2 <))
(define (medium l) (sized l 5 =))
```

Sized Fish

```
; sized : list-of-nums num (num num -> bool)
; -> list-of-nums
(define (sized l n COMP)
  (cond
    [(empty? l) empty]
    [(cons? l)
     (local [(define r
               (sized (rest l) n COMP))]
       (cond
         [(COMP (first l) n)
          (cons (first l) r)]
         [else r]))]))
```

How about all fish between 3 and 7 lbs?

Mediumish Fish

```
; btw-3-and-7 : num num -> bool
(define (btw-3-and-7 a ignored-zero)
  (and (>= a 3)
       (<= a 7)))

(define (mediumish l) (sized l 0 btw-3-and-7))
```

Mediumish Fish

```
; btw-3-and-7 : num num -> bool
(define (btw-3-and-7 a ignored-zero)
  (and (>= a 3)
       (<= a 7)))

(define (mediumish l) (sized l 0 btw-3-and-7))
```

- Programmer-defined functions are values, too
- Note that the contract of **`btw-3-and-7`** matches the kind expected by **`sized`**

Mediumish Fish

```
; btw-3-and-7 : num num -> bool
(define (btw-3-and-7 a ignored-zero)
  (and (>= a 3)
       (<= a 7)))

(define (mediumish l) (sized l 0 btw-3-and-7))
```

- Programmer-defined functions are values, too
- Note that the contract of **`btw-3-and-7`** matches the kind expected by **`sized`**

But the ignored **`0`** suggests a simplification of **`sized`**...

A Generic Number Filter

```
; filter-nums : (num -> bool) list-of-num
; -> list-of-num
(define (filter-nums PRED l)
  (cond
    [(empty? l) empty]
    [(cons? l)
     (local [(define r
               (filter-nums PRED (rest l)))]
       (cond
         [(PRED (first l))
          (cons (first l) r)]
         [else r])))]))
```

A Generic Number Filter

```
; filter-nums : (num -> bool) list-of-num
; -> list-of-num
(define (filter-nums PRED l)
  (cond
    [(empty? l) empty]
    [(cons? l)
     (local [(define r
               (filter-nums PRED (rest l)))]
       (cond
         [(PRED (first l))
          (cons (first l) r)]
         [else r])))]))

(define (btw-3&7 n) (and (>= n 3) (<= n 7)))
(define (mediumish l) (filter-nums btw-3&7 l))
```

Big and Huge Fish, Again

```
(define (more-than-5 n)
  (> n 5))
(define (big l)
  (filter-nums more-than-5 l))

(define (more-than-10 n)
  (> n 10))
(define (huge l)
  (filter-nums more-than-10 l))
```

Big and Huge Fish, Again

```
(define (more-than-5 n)
  (> n 5))
(define (big l)
  (filter-nums more-than-5 l))

(define (more-than-10 n)
  (> n 10))
(define (huge l)
  (filter-nums more-than-10 l))
```

The **`more-than-5`** and **`more-than-10`** functions are really only useful to **`big`** and **`huge`**

We could make them **`local`** to clarify...

Big and Huge Fish, Improved

```
(define (big l)
  (local [(define (more-than-5 n)
                  (> n 5))]
    (filter-nums more-than-5 l)))

(define (huge l)
  (local [(define (more-than-10 n)
                  (> n 10))]
    (filter-nums more-than-10 l)))
```

Big and Huge Fish, Improved

```
(define (big l)
  (local [(define (more-than-5 n)
                  (> n 5))]
    (filter-nums more-than-5 l)))

(define (huge l)
  (local [(define (more-than-10 n)
                  (> n 10))]
    (filter-nums more-than-10 l)))
```

Cut and paste alert!

You don’t think I typed that twice, do you?

Big and Huge Fish, Generalized

```
(define (bigger-than l m)
  (local [(define (more-than-m n)
                 (> n m))]
    (filter-nums more-than-m l)))

(define (big l) (bigger-than l 5))
(define (huge l) (bigger-than l 10))
```

Big Example

```
...
(define (bigger-than l m)
  (local [(define (more-than-m n)
            (> n m))]
    (filter-nums more-than-m l)))
(define (big l) (bigger-than l 5)) ...
(big '(7 4 9))
(huge '(7 4 9))
```

Big Example

```
...
(define (bigger-than l m)
  (local [(define (more-than-m n)
            (> n m))]
    (filter-nums more-than-m l)))
(define (big l) (bigger-than l 5)) ...
(big '(7 4 9))
(huge '(7 4 9))
```

$\rightarrow$

```
...
(define (bigger-than l m)
  (local [(define (more-than-m n)
            (> n m))]
    (filter-nums more-than-m l)))
...
(bigger-than '(7 4 9) 5)
(huge '(7 4 9))
```

Big Example

```
...
(define (bigger-than l m)
  (local [(define (more-than-m n)
              (> n m))]
    (filter-nums more-than-m l)))
...
(bigger-than '(7 4 9) 5)
(huge '(7 4 9))
```

Big Example

```
...
(define (bigger-than l m)
  (local [(define (more-than-m n)
            (> n m))]
    (filter-nums more-than-m l)))
...
(bigger-than '(7 4 9) 5)
(huge '(7 4 9))
```

$\rightarrow$

```
...
(local [(define (more-than-m n)
          (> n 5))]
  (filter-nums more-than-m '(7 4 9)))
(huge '(7 4 9))
```

Big Example

```
...
(local [(define (more-than-m n)
          (> n 5))]
  (filter-nums more-than-m '(7 4 9)))
(huge '(7 4 9))
```

Big Example

```
...
(local [(define (more-than-m n)
          (> n 5))]
  (filter-nums more-than-m '(7 4 9)))
(huge '(7 4 9))
```

$\rightarrow$

```
...
(define (more-than-m42 n)
  (> n 5))
(filter-nums more-than-m42 '(7 4 9))
(huge '(7 4 9))
```

Big Example

```
...
(define (more-than-m42 n)
  (> n 5))
(filter-nums more-than-m42 '(7 4 9))
(huge '(7 4 9))
```

Big Example

```
...
(define (more-than-m42 n)
  (> n 5))
(filter-nums more-than-m42 '(7 4 9))
(huge '(7 4 9))
```

→

```
...
(define (more-than-m42 n)
  (> n 5))
'(7 9)
(huge '(7 4 9))
```

after many steps

Big Example

```
...
(define (more-than-m42 n)
  (> n 5))
'(7 9)
(huge '(7 4 9))
```

Big Example

```
...
(define (more-than-m42 n)
  (> n 5))
'(7 9)
(huge '(7 4 9))
```

$\rightarrow$

```
...
(define (bigger-than l m)
  (local [(define (more-than-m n)
            (> n m))]
    (filter-nums more-than-m l)))
...
(define (more-than-m42 n)
  (> n 5))
'(7 9)
(bigger-than '(7 4 9) 10)
```

Big Example

```
...
(define (bigger-than l m)
  (local [(define (more-than-m n)
               (> n m))]
    (filter-nums more-than-m l)))
...
(define (more-than-m42 n)
  (> n 5))
'(7 9)
(bigger-than '(7 4 9) 10)
```

Big Example

```
...
(define (bigger-than l m)
  (local [(define (more-than-m n)
               (> n m))]
    (filter-nums more-than-m l)))
...
(define (more-than-m42 n)
  (> n 5))
'(7 9)
(bigger-than '(7 4 9) 10)
```

$\rightarrow$

```
...
(define (more-than-m42 n)
  (> n 5))
'(7 9)
(local [(define (more-than-m n)
             (> n 10))]
  (filter-nums more-than-m '(7 4 9)))
```

Big Example

```
...
(define (more-than-m42 n)
  (> n 5))
'(7 9)
(local [(define (more-than-m n)
          (> n 10))]
  (filter-nums more-than-m '(7 4 9)))
```

Big Example

```
...
(define (more-than-m42 n)
  (> n 5))
'(7 9)
(local [(define (more-than-m n)
          (> n 10))]
  (filter-nums more-than-m '(7 4 9)))
```

$\rightarrow$

```
...
(define (more-than-m42 n)
  (> n 5))
'(7 9)
(define (more-than-m79 n)
  (> n 10))
(filter-nums more-than-m79 '(7 4 9))
```

Etc.

Abstraction

- Avoiding cut and paste is ***abstraction***
- No real programming task succeeds without it

- **Function Abstraction**
- **Type Abstraction**
- **Anonymous Functions**

Symbols

Our favorite **`list-of-sym`** program:

```
; eat-apples : list-of-sym -> list-of-sym
(define (eat-apples l)
  (cond
   [(empty? l) empty]
   [(cons? l)
    (local [(define ate-rest (eat-apples (rest l)))]
      (cond
        [(symbol=? (first l) 'apple) ate-rest]
        [else (cons (first l) ate-rest)]))]))
```

- How about **`eat-bananas`**?
- How about **`eat-non-apples`**?

Symbols

Our favorite **`list-of-sym`** program:

```
; eat-apples : list-of-sym -> list-of-sym
(define (eat-apples l)
  (cond
   [(empty? l) empty]
   [(cons? l)
    (local [(define ate-rest (eat-apples (rest l)))]
      (cond
        [(symbol=? (first l) 'apple) ate-rest]
        [else (cons (first l) ate-rest)]))]))
```

- How about **`eat-bananas`**?
- How about **`eat-non-apples`**?

We know where this leads...

Filtering Symbols

```
; filter-syms : (sym -> bool) list-of-sym
; -> list-of-sym
(define (filter-syms PRED l)
  (cond
    [(empty? l) empty]
    [(cons? l)
     (local [(define r
               (filter-syms PRED (rest l)))]
       (cond
         [(PRED (first l))
          (cons (first l) r)]
         [else r])))]))
```

Filtering Symbols

```
; filter-syms : (sym -> bool) list-of-sym
; -> list-of-sym
(define (filter-syms PRED l)
  (cond
    [(empty? l) empty]
    [(cons? l)
     (local [(define r
               (filter-syms PRED (rest l)))]
       (cond
         [(PRED (first l))
          (cons (first l) r)]
         [else r])))]))
```

This looks *really* familiar

Last Time: Filtering Numbers

```
; filter-nums : (num -> bool) list-of-num
; -> list-of-num
(define (filter-nums PRED l)
  (cond
    [(empty? l) empty]
    [(cons? l)
     (local [(define r
               (filter-nums PRED (rest l)))]
       (cond
         [(PRED (first l))
          (cons (first l) r)]
         [else r])))]))
```

Last Time: Filtering Numbers

```
; filter-nums : (num -> bool) list-of-num
; -> list-of-num
(define (filter-nums PRED l)
  (cond
    [(empty? l) empty]
    [(cons? l)
     (local [(define r
               (filter-nums PRED (rest l)))]
       (cond
         [(PRED (first l))
          (cons (first l) r)]
         [else r])))]))
```

How do we avoid cut and paste?

Filtering Lists

We know this function will work for both number and symbol lists:

```
; filter : ...
(define (filter PRED l)
  (cond
    [(empty? l) empty]
    [(cons? l)
     (local [(define r
                (filter PRED (rest l)))]
       (cond
         [(PRED (first l))
          (cons (first l) r)]
         [else r])))]))
```

But what is its contract?

The Contract of Filter

How about this?

```
(num-OR-sym -> bool) list-of-num-OR-list-of-sym
-> list-of-num-OR-list-of-sym

 ; A num-OR-sym is either
 ;  - num
 ;  - sym

 ; A list-of-num-OR-list-of-sym is either
 ;  - list-of-num
 ;  - list-of-sym
```

The Contract of Filter

How about this?

```
(num-OR-sym -> bool) list-of-num-OR-list-of-sym
-> list-of-num-OR-list-of-sym
```

This contract is too weak to define **`eat-apples`**

```
; eat-apples : list-of-sym -> list-of-sym
(define (eat-apples l)
  (filter not-apple? l))

; not-apple? : sym -> bool
(define (not-apple? s)
  (not (symbol=? s 'apple)))
```

`eat-apples` must return a **`list-of-sym`**, but by its contract, **`filter`** might return a **`list-of-num`**

The Contract of Filter

How about this?

```
(num-OR-sym -> bool) list-of-num-OR-list-of-sym
-> list-of-num-OR-list-of-sym
```

This contract is too weak to define **`eat-apples`**

```
; eat-apples : list-of-sym -> list-of-sym
(define (eat-apples l)
  (filter not-apple? l))

; not-apple? : sym -> bool
(define (not-apple? s)
  (not (symbol=? s 'apple)))
```

`not-apple?` only works on symbols, but by its contract **`filter`** might give it a **`num`**

The Contract of Filter

The reason **`filter`** works is that if we give it a **`list-of-sym`**, then it returns a **`list-of-sym`**

Also, if we give **`filter`** a **`list-of-sym`**, then it calls **`PRED`** with symbols only

The Contract of Filter

The reason **`filter`** works is that if we give it a **`list-of-sym`**, then it returns a **`list-of-sym`**

Also, if we give **`filter`** a **`list-of-sym`**, then it calls **`PRED`** with symbols only

A better contract:

```
filter :
((num -> bool) list-of-num
 -> list-of-num)
OR
((sym -> bool) list-of-sym
 -> list-of-sym)
```

The Contract of Filter

The reason **`filter`** works is that if we give it a **`list-of-sym`**, then it returns a **`list-of-sym`**

Also, if we give **`filter`** a **`list-of-sym`**, then it calls **`PRED`** with symbols only

A better contract:

```
filter :
((num -> bool) list-of-num
 -> list-of-num)
OR
((sym -> bool) list-of-sym
 -> list-of-sym)
```

But what about a list of **`image`**s, **`posn`**s, or **`snake`**s?

The True Contract of Filter

The real contract is

```
filter : ((X -> bool) list-of-X -> list-of-X)
```

where **`X`** stands for any type

- The caller of **`filter`** gets to pick a type for **`X`**
- All **`X`**s in the contract must be replaced with the same type

The True Contract of Filter

The real contract is

```
filter : ((X -> bool) list-of-X -> list-of-X)
```

where **`X`** stands for any type

- The caller of **`filter`** gets to pick a type for **`X`**
- All **`X`**s in the contract must be replaced with the same type

Data definitions need type variables, too:

```
; A list-of-X is either
;   - empty
;   - (cons X empty)
```

Using Filter

The **`filter`** function is so useful that it's built in

New solution:

```
(define (eat-apples l)
  (local [(define (not-apple? s)
            (not (symbol=? s 'apple)))]
    (filter not-apple? l)))
```

Looking for Other Built-In Functions

Recall **`inflate-by-4%`**:

```
; inflate-by-4% : list-of-num -> list-of-num
(define (inflate-by-4% l)
  (cond
    [(empty? l) empty]
    [else (cons (* (first l) 1.04)
                (inflate-by-4% (rest l)))]))
```

Is there a built-in function to help?

Looking for Other Built-In Functions

Recall **inflate-by-4%**:

```
; inflate-by-4% : list-of-num -> list-of-num
(define (inflate-by-4% l)
  (cond
    [(empty? l) empty]
    [else (cons (* (first l) 1.04)
                (inflate-by-4% (rest l)))]))
```

Is there a built-in function to help?

Yes: **map**

Using Map

```
(define (map CONV l)
  (cond
    [(empty? l) empty]
    [else (cons (CONV (first l))
                (map CONV (rest l)))]))


(define (inflate-by-4% l)
  (local [(define (inflate-one n)
            (* n 1.04))]
    (map inflate-one l)))

; negate-colors : list-of-col -> list-of-col
(define (negate-colors l)
  (map negate-color l))
```

The Contract for Map

```
(define (map CONV l)
  (cond
    [(empty? l) empty]
    [else (cons (CONV (first l))
                (map CONV (rest l)))]))
```

- The `l` argument must be a list of `X`
- The `CONV` argument must accept each `X`
- If `CONV` returns a new `X` each time, then the contract for `map` is

```
map : (X -> X) list-of-X -> list-of-X
```

Posns and Distances

Another function from HW 4:

```
; distances : list-of-posn -> list-of-num
(define (distances l)
  (cond
    [(empty? l) empty]
    [(cons? l) (cons (distance-to-0 (first l))
                     (distances (rest l)))]))
```

Posns and Distances

Another function from HW 4:

```
; distances : list-of-posn -> list-of-num
(define (distances l)
  (cond
    [(empty? l) empty]
    [(cons? l) (cons (distance-to-0 (first l))
                     (distances (rest l)))]))
```

The **`distances`** function looks just like **`map`**, except that **`distances-to-0`** is

`posn -> num`

not

`posn -> posn`

The True Contract of Map

Despite the contract mismatch, this works!

```
(define (distances l)
  (map distance-to-0 l))
```

The True Contract of Map

Despite the contract mismatch, this works!

```
(define (distances l)
  (map distance-to-0 l))
```

The true contract of **map** is

```
map : (X -> Y) list-of-X -> list-of-Y
```

The caller gets to pick both **X** and **Y** independently

More Uses of Map

```
; modernize : list-of-pipe -> list-of-pipe
(define (modernize l)
  ; replaces 4 lines:
  (map modern-pipe l))

; modern-pipe : pipe -> pipe
...

; rob-train : list-of-car -> list-of-car
(define (rob-train l)
  ; replaces 4 lines:
  (map rob-car l))

; rob-car : car -> car
...
```

Folding a List

How about **`sum`**?

```
sum : list-of-num -> num
```

Doesn’t return a list, so neither **`filter`** nor **`map`** help

Folding a List

How about **sum**?

sum : list-of-num -> num

Doesn't return a list, so neither **filter** nor **map** help

But recall **combine-nums**...

```
; combine-nums : list-of-num num
; (num num -> num) -> num
(define (combine-nums l base-n COMB)
  (cond
    [(empty? l) base-n]
    [(cons? l)
     (COMB
      (first l)
      (combine-nums (rest l) base-n COMB))]))
```

The Foldr Function

```
; foldr : (X Y -> Y) Y list-of-X -> Y
(define (foldr COMB base l)
  (cond
    [(empty? l) base]
    [(cons? l)
     (COMB (first l)
           (foldr COMB base (rest l)))]))
```

The Foldr Function

```
; foldr : (X Y -> Y) Y list-of-X -> Y
(define (foldr COMB base l)
  (cond
    [(empty? l) base]
    [(cons? l)
     (COMB (first l)
           (foldr COMB base (rest l)))]))
```

The **sum** and **product** functions become trivial:

```
(define (sum l) (foldr + 0 l))
(define (product l) (foldr * 1 l))
```

The Foldr Function

```
; foldr : (X Y -> Y) Y list-of-X -> Y
(define (foldr COMB base l)
  (cond
    [(empty? l) base]
    [(cons? l)
     (COMB (first l)
           (foldr COMB base (rest l)))]))
```

Useful for HW 5:

```
; total-blue : list-of-col -> num
(define (total-blue l)
  (local [(define (add-blue c n)
            (+ (color-blue c) n))]
    (foldr add-blue 0 l)))
```

The Foldr Function

```
; foldr : (X Y -> Y) Y list-of-X -> Y
(define (foldr COMB base l)
  (cond
    [(empty? l) base]
    [(cons? l)
     (COMB (first l)
           (foldr COMB base (rest l)))]))
```

In fact,

```
(define (map f l)
  (local [(define (comb i r)
            (cons (f i) r))]
    (foldr comb empty l)))
```

The Foldr Function

```
; foldr : (X Y -> Y) Y list-of-X -> Y
(define (foldr COMB base l)
  (cond
    [(empty? l) base]
    [(cons? l)
     (COMB (first l)
           (foldr COMB base (rest l)))]))
```

Yes, **filter** too:

```
(define (filter f l)
  (local [(define (check i r)
            (cond
              [(f i) (cons i r)]
              [else r]))]
    (foldr check empty l)))
```

The Source of Foldr

How can **`foldr`** be so powerful?

The Source of Foldr

Template:

```
(define (func-for-loX l)
  (cond
    [(empty? l) ...]
    [(cons? l) ... (first l)
     ... (func-for-loX (rest l)) ...]))
```

Fold:

```
(define (foldr COMB base l)
  (cond
    [(empty? l) base]
    [(cons? l)
     (COMB (first l)
           (foldr COMB base (rest l)))]))
```

Other Built-In List Functions

More specializations of **`foldr`**:

```
ormap : (X -> bool) list-of-X -> bool

andmap : (X -> bool) list-of-X -> bool
```

Examples:

```
; got-milk? : list-of-sym -> bool
(define (got-milk? l)
  (local [(define (is-milk? s)
            (symbol=? s 'milk))]
    (ormap is-milk? s)))

; all-passed? : list-of-grade -> bool
(define (all-passed? l)
  (andmap passing-grade? l))
```

What about Non-Lists?

Since it's based on the template, the concept of fold is general

```
; fold-ftn : (sym num sym Z Z -> Z) Z ftn -> Z
(define (fold-ftn COMB base ftn)
  (cond
    [(empty? ftn) base]
    [(child? ftn)
     (COMB (child-name ftn) (child-date ftn) (child-eyes ftn)
           (fold-ftn COMB BASE (child-father ftn))
           (fold-ftn COMB BASE (child-mother ftn)))]))

(define (count-persons ftn)
  (local [(define (add name date color c-f c-m)
            (+ 1 c-f c-m))]
    (fold-ftn add 0 ftn)))

(define (in-family? who ftn)
  (local [(define (here? name date color in-f? in-m?)
            (or (symbol=? name who) in-f? in-m?))]
    (fold-ftn here? false ftn)))
```

- **Function Abstraction**
- **Type Abstraction**
- **Anonymous Functions**

Values and Names

Some Values:

- Numbers: **`1`**, **`17.8`**, **`4/5`**
- Booleans: **`true`**, **`false`**
- Lists: **`empty`**, **`(cons 7 empty)`**
- ...
- Function *names*: **`less-than-5`**, **`first-is-apple?`**

  given

```
(define (less-than-5? n) ...)
(define (first-is-apple? a b) ...)
```

Values and Names

Some Values:

- Numbers: `1`, `17.8`, `4/5`
- Booleans: `true`, `false`
- Lists: `empty`, `(cons 7 empty)`
- ...
- Function *names*: `less-than-5`, `first-is-apple?`

  given

```
(define (less-than-5? n) ...)
(define (first-is-apple? a b) ...)
```

Why do only function values require names?

Naming Everything

Having to name *every* kind of value would be painful:

```
(local [(define (first-is-apple? a b)
          (symbol=? a 'apple))]
  (choose '(apple banana) '(cherry cherry)
          first-is-apple?))
```

whould have to be

```
(local [(define (first-is-apple? a b)
          (symbol=? a 'apple))
        (define al '(apple banana))
        (define bl '(cherry cherry))]
  (choose al bl first-is-apple?))
```

Fortunately, we don't have to name lists

Naming Nothing

Can we avoid naming functions?

In other words, instead of writing

```
(local [(define (first-is-apple? a b)
          (symbol=? a 'apple))]
  ... first-is-apple? ...)
```

we'd like to write

...
  function that takes **a** and **b**
    and produces **(symbol=? a 'apple)**
...

Naming Nothing

Can we avoid naming functions?

In other words, instead of writing

```
(local [(define (first-is-apple? a b)
          (symbol=? a 'apple))]
  ... first-is-apple? ...)
```

we'd like to write

...

function that takes **a** and **b**

and produces `(symbol=? a 'apple)`

...

We can do this

Lambda

An ***anonymous function*** value:

```
(lambda (a b) (symbol=? a 'apple))
```

Using **`lambda`** the original example becomes

```
(choose '(apple banana) '(cherry cherry)
        (lambda (a b) (symbol=? a 'apple)))
```

Lambda

An ***anonymous function*** value:

```
(lambda (a b) (symbol=? a 'apple))
```

Using **lambda** the original example becomes

```
(choose '(apple banana) '(cherry cherry)
        (lambda (a b) (symbol=? a 'apple)))
```

Why the funny keyword **lambda**?

It's a 70-year-old convention: the Greek letter λ means "function"

Using Lambda

In DrScheme:

```
> (lambda (x) (+ x 10))
(lambda (a1) ...)
```

Unlike most kinds of values, there's no one shortest name:

- The argument name is arbitrary
- The body can be implemented in many different ways

So DrScheme gives up — it invents argument names and hides the body

Using Lambda

In DrScheme:

```
> ((lambda (x) (+ x 10)) 17)
27
```

The function position of an ***application*** (i.e., function call) is no longer always an identifier

Using Lambda

In DrScheme:

```
> ((lambda (x) (+ x 10)) 17)
27
```

The function position of an ***application*** (i.e., function call) is no longer always an identifier

Some former syntax errors are now run-time errors:

```
> (2 3)
```

procedure application: expected procedure, given 2

Defining Functions

What's the difference between

```
(define (f a b)
  (+ a b))
```

and

```
(define f (lambda (a b)
            (+ a b)))
```

?

Defining Functions

What's the difference between

```
(define (f a b)
  (+ a b))
```

and

```
(define f (lambda (a b)
            (+ a b)))
```

?

Nothing — the first one is (now) a shorthand for the second

Lambda and Built-In Functions

Anonymous functions work great with **`filter`**, **`map`**, etc.:

```
(define (eat-apples l)
  (filter (lambda (a)
            (not (symbol=? a 'apple)))
          l))

(define (inflate-by-4% l)
  (map (lambda (n) (* n 1.04)) l))

(define (total-blue l)
  (foldr (lambda (c n)
           (+ (color-blue c) n))
         0 l))
```

Functions that Produce Functions

We already have functions that take function arguments

```
map : (X -> Y) list-of-X -> list-of-Y
```

How about functions that *produce* functions?

Functions that Produce Functions

We already have functions that take function arguments

```
map : (X -> Y) list-of-X -> list-of-Y
```

How about functions that *produce* functions?

Here's one:

```
; make-adder : num -> (num -> num)
(define (make-adder n)
  (lambda (m) (+ m n)))

(map (make-adder 10) '(1 2 3))
(map (make-adder 11) '(1 2 3))
```

Using Functions that Produce Functions

Suppose that we need to filter different symbols:

```
(filter (lambda (a) (symbol=? a 'apple)) l)
(filter (lambda (a) (symbol=? a 'banana)) l)
(filter (lambda (a) (symbol=? a 'cherry)) l)
```

Using Functions that Produce Functions

Suppose that we need to filter different symbols:

```
(filter (lambda (a) (symbol=? a 'apple)) l)
(filter (lambda (a) (symbol=? a 'banana)) l)
(filter (lambda (a) (symbol=? a 'cherry)) l)
```

Instead of repeating the long **`lambda`** expression, we can abstract:

```
; mk-is-sym : sym -> (sym -> bool)
(define (mk-is-sym s)
  (lambda (a) (symbol=? s a)))

(filter (mk-is-sym 'apple) l)
(filter (mk-is-sym 'banana) l)
(filter (mk-is-sym 'cherry) l)
```

Using Functions that Produce Functions

Suppose that we need to filter different symbols:

```
(filter (lambda (a) (symbol=? a 'apple)) l)
(filter (lambda (a) (symbol=? a 'banana)) l)
(filter (lambda (a) (symbol=? a 'cherry)) l)
```

Instead of repeating the long **lambda** expression, we can abstract:

```
; mk-is-sym : sym -> (sym -> bool)
(define (mk-is-sym s)
  (lambda (a) (symbol=? s a)))

(filter (mk-is-sym 'apple) l)
(filter (mk-is-sym 'banana) l)
(filter (mk-is-sym 'cherry) l)
```

mk-is-sym is a ***curried*** version of **symbol=?**

! Currying Functions !

This **curry** function curries any 2-argument function:

```
; curry : (X Y -> Z) -> (X -> (Y -> Z))
(define (curry f)
  (lambda (v1)
    (lambda (v2)
      (f v1 v2))))

(define mk-is-sym (curry symbol=?))

(filter (mk-is-sym 'apple) l)
(filter (mk-is-sym 'banana) l)
(filter (mk-is-sym 'cherry) l)
```

! Currying Functions !

This **curry** function curries any 2-argument function:

```
; curry : (X Y -> Z) -> (X -> (Y -> Z))
(define (curry f)
  (lambda (v1)
    (lambda (v2)
      (f v1 v2))))

(filter ((curry symbol=?) 'apple) l)
(filter ((curry symbol=?) 'banana) l)
(filter ((curry symbol=?) 'cherry) l)
```

! Composing Functions !

But we want *non*-symbols

```
; compose (Y -> Z) (X ->Y) -> (X -> Z)
(define (compose f g)
  (lambda (x) (f (g x))))

(filter (compose
         not
         ((curry symbol=?) 'apple))
        l)
```

! Uncurrying Functions !

Sometimes it makes sense to ***uncurry***:

```
; curry : (X -> (Y -> Z)) -> (X Y -> Z)
(define (uncurry f)
   (lambda (v1 v2)
     ((f v1) v2)))

(define (map f l)
  (foldr (uncurry (compose (curry cons) f))
         empty l))

(define (total-blue l)
  (foldr (uncurry (compose (curry +)
                           color-blue))
         0 l))
```

Lambda in Math

```
; derivative : (num -> num) -> (num -> num)
(define (derivative f)
  (lambda (x)
    (/ (- (f (+ x delta))
          (f (- x delta)))
       (* 2 delta))))
(define delta 0.0001)

(define (square n) (* n n))
((derivative square) 10)
```

Produces roughly **20**, because the derivative of x^2 is $2x$

Lambda in Real Life

Graphical User Interfaces (GUIs) often use functions as values, including anonymous functions

Java equivalent: inner classes

Button click $\Rightarrow$ update bottom text

GUI Library

```
make-text : string -> gui-item

text-contents : gui-item -> string


make-message : string -> gui-item

draw-message : gui-item string -> bool


make-button : string (event -> bool) -> gui-item


create-window : list-of-list-of-gui-item -> bool
```

GUI Example

```
(define (greet what)
  (draw-message greet-msg
                (string-append
                 what ", "
                 (text-contents name-field))))

(define name-field
  (make-text "Name:"))
(define hi-button
  (make-button "Hello" (lambda (evt) (greet "Hi"))))
(define bye-button
  (make-button "Goodbye" (lambda (evt) (greet "Bye"))))
(define greet-msg
  (make-message "__________________________"))
```

GUI Example Improved

```
(define (mk-greet what)
  (lambda (evt)
    (draw-message greet-msg
                  (string-append
                   what ", "
                   (text-contents name-field)))))
(define name-field
  (make-text "Name:"))
(define hi-button
  (make-button "Hello" (mk-greet "Hi")))
(define bye-button
  (make-button "Goodbye" (mk-greet "Bye")))
(define greet-msg
  (make-message "__________________________"))
```